Rationale of the 'irrational' number

'Irrational numbers', mathematicians' delight and despair, may come to the aid of e-commerce, says **S.Ananthanarayanan**

Then what is a 'rational' number?

All fractions, or ratios of integers, like 9/8 or 143/54, for example, can be expressed in decimal notation either as a 'terminating' or a 'recurring' decimal. Here, 9/8=1.125 stops at the 5, but in 1/3=.33333, the '3's go on endlessly. In the number 9/7=1.28571428…., the group of 6 digits after the decimal point repeat endlessly. The digits stop if the division gives a zero remainder, or repeat when the remainder is the same as any previous remainder.

Such decimal numbers, which are all finally the value of some ratio of integers, are called 'rational numbers,' or numbers that are a 'ratio' of integers.

And Irrational numbers?

In these too, the number goes on and on, like a recurring decimal, except that the digits do not repeat. You can see that such a number could not be the ratio of two integers. For if it could, then that ratio would evaluate to a 'terminating' or a 'recurring' decimal!

Such numbers arise when dealing with curved shapes, square roots, adding infinite series, and stuff like that

The square root of 2 is one such. This number can be 'drawn', as the diagonal of square with sides equal to 1, but it can be shown quite easily
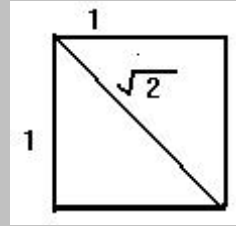
that this value cannot be put in the form p/q, where p and q are integers, which means, as we said, that it is 'irrational'.

The other classic instance is pi, whose value is given by the series: 4–4/3+4/5–4/7+4/9–4/1..±4/(successive odd numbers)…. to infinite terms. If we took only the first 2 terms, we have 4-1.33=2.67. With the third term, we add 4/5 and get 3.47. Another term and it is 3.47-4/7=2.90. Yet another and we have 3.02+4/9=3.34, 2.98, 3.28, 3.02, etc.

We can see that it approaches the well known, but approximate, value of 3.14, as we consider more terms. Considering thousands of terms, which is possible with a computer, takes us to very accurate values indeed!

Computer codes

Mathematicians find great beauty and significance in the properties of irrational numbers and the study has become complex and esoteric. But nearer home, the detailed value of pi may be the solution that makers of computer codes are looking for.

Many computer codes depend on sequences of random numbers. But we do not as yet have a truly 'random' method of generating random numbers and the algorithms used can all be cracked.

But the infinite and non-recurring expansion of pi, it turns out, may be the solution. If we expanded pi to a million places and selected a digit somewhere in the string, there is just no way to decode what the digits that follow would be, as every digit appears, quite differently, so many times in the mass of a million digits!

If a code were based on some number in the expansion and we passed on that starting point to our correspondent, then she could continue using successive numbers, which eavesdroppers could not guess!

Value of pi

A mnemonic for pi to 11 digits goes like this:

"Now I know a rhyme excelling, of mystic words and magic spelling."

The number of letters in the words of this mnemonic spell them out:

3.14159265358

Computers, of course, can do much better:

Pi = 3.1415926535 8979323846 2643383279
5028841971   6939937510   035820974944
5923078164 0628620899 86284825 421170679
[Pi to 100 decimal places]

Pi = 3.1415926535 8979323846 2643383279 5028841971 6939937510
5820974944 5923078164 0628620899 8628034825 3421170679
8214808651 3282306647 0938446095 5058223172 5359408128
4811174502 8410270193 8521105559 6446229489 5493038196
[pi to 200 decimal places]