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Special places in satellite orbit

The effect of gravity of two heavenly bodies on a third object could sometimes cancel out, says S.Ananthanrayanan.

This kind of problem, where there are in fact 3 objects in motion and all have gravitational effects is called the 'three body problem' and has been a difficult conundrum to solve, ever since Newton solved at least the 2 body problem, of a planet and its satellite.

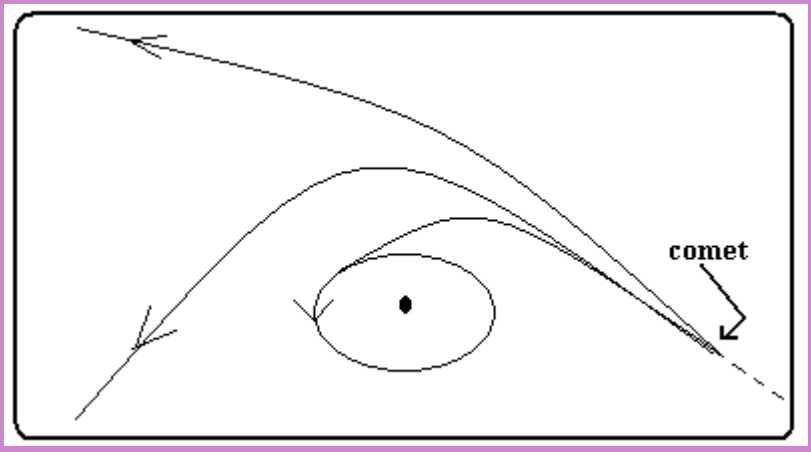
Work of Lagrange

Joseph Louis Lagrange was a French-Italian mathematician who developed a whole new way of working with dynamical systems. The usual way is to use the position of a particle and the property that its total energy, both due to attraction to other objects and due to its motion, should be constant, to help work out the path the particle would follow.

Lagrange took the two parts of the energy of the particle, due to attraction and due to motion, and said the path the particle followed was the one along which the total difference of the two energies, over the entire path of motion, stayed the least! It is possible to use the usual way of looking at things and to prove that this new way is also true, but this new way is very powerful in solving some problems that become tiresome indeed in the usual way.

Simple Three body

A simple case of the three body problem is when the 3 rd body has definite, but negligible mass. This kind of body is attracted by the other two, but does not have any significant effect on them. Even this problem is interesting because the two massive objects also do not stay put but move because of their own initial motion, and the effect of the other massive body.



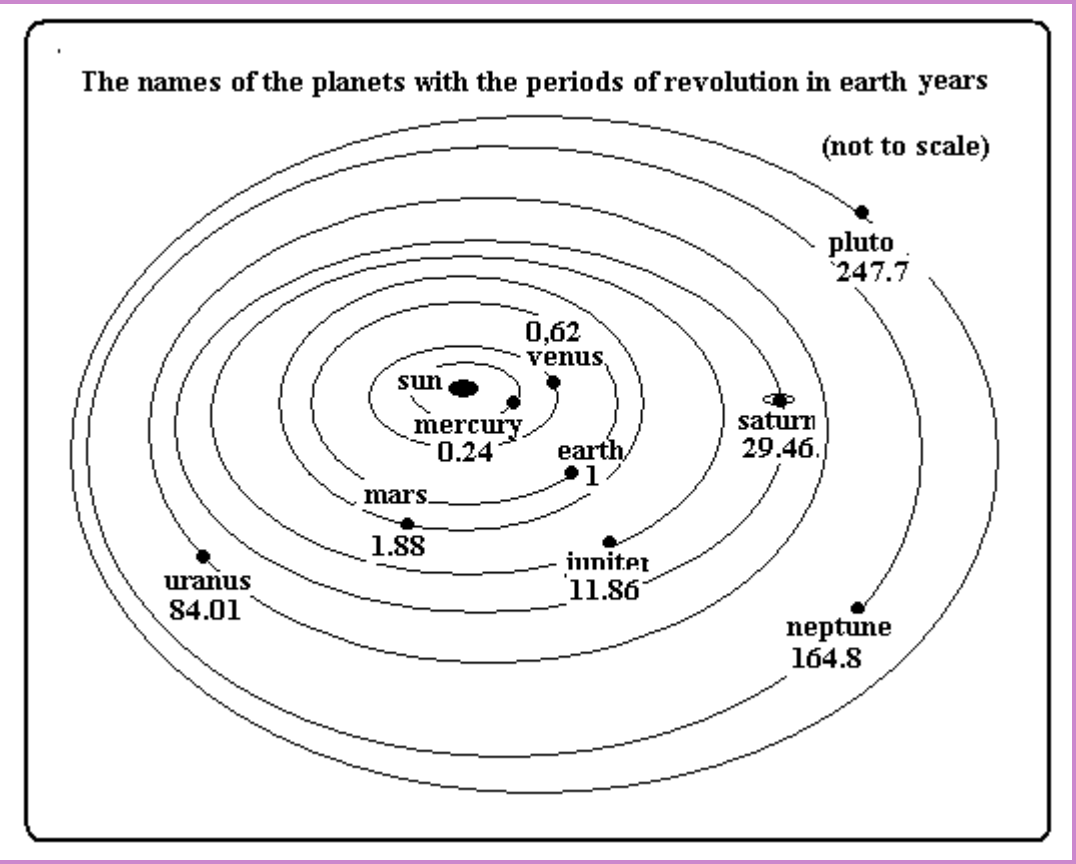
For example, if a comet shoots past a planet, the path of the comet is curved because of the planet and the planet gets slightly affected because of the comet, like in the picture. If the comet is not moving too fast, it may even get 'captured' and fall into a circular or elliptical orbit.

An interesting type of the simpler three body problem arises when a satellite goes around a massive body and a third object of negligible mass is also affected by the third object, or even if it is in orbit around the same object.

The mechanics of orbits

When things move around in circles, they feel a force that drives them away from the centre – the centrifugal force. Now, when a comet approaches a planet or star, it begins to curve and immediately feels this force. This force keeps acting against the attraction towards the centre and the comet can eventually get away. Even if does not, it can get into an orbit around the centre, always moving faster if it gets closer, so that it counteracts the attraction.

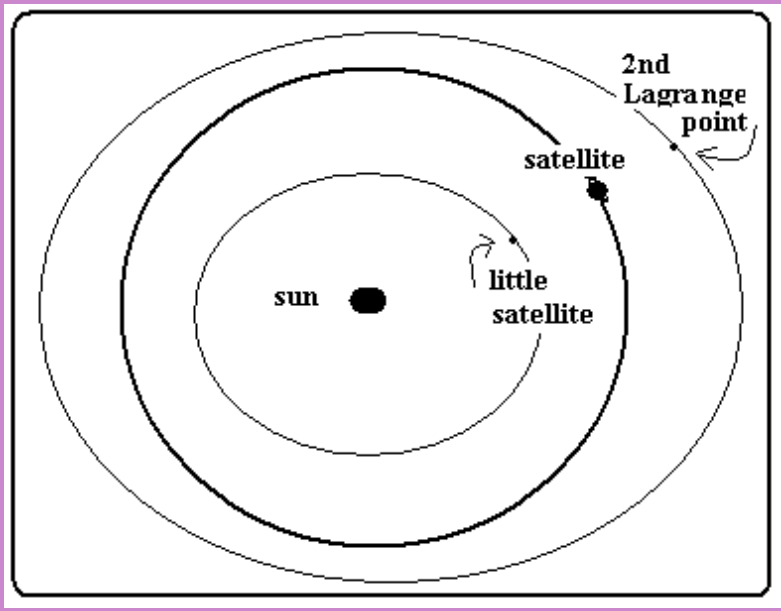
A satellite, in fact, needs to move faster when it is closer to the centre. This is nicely demonstrated in the time the planets of the solar system take to go around the sun. As shown in the picture, the innermost planets move the fastest.



Now let us take the case of just two planets going round a sun, with the inner planet being of 'negligible' mass, like in the next picture. It is easy to see that the inner planet will move faster and go round in less time than the outer planet. And that this is because the inner planet experiences more gravity than the outer one. But then, when the planets are almost in line, the inner planet also feels the outward pull of the outer planet. The net inward pull is thus less and the planet would slow down.

Lagrange Points

Lagrange worked it out that for a particular distance between the star and the planets, the inner planet would go round the star in the same time as the outer planet. This is because the gravity at the inner orbit, due to the effect of the outer planet, is the same as at the outer orbit. This point, in honour of the mathematician, is called the ***Lagrange point***.



A similar point also exists outside the orbit of a planet. In a larger orbit, the force of gravity is less and the orbit should be slower. But if the planets are in line, then the inner planet adds to the force of gravity and the total force can be equal to the force at the inner orbit. Then the outer planet will also follow the inner planet, like a sheepdog. This point is called the 2nd Lagrange point.

Lagrange points are important for placing artificial satellites or orbiting telescopes, so that they keep time with the earth and the moon or to be protected from varying gravity fields.

Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto	
mean distance from Sun (AU)	0.39	0.72	1	1.52	5.20	9.54	19.18	30.06	39.44
orbital period (Earth years)	0.24	0.62	1	1.88	11.86	29.46	84.01	164.8	247.7