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Mathematics and Astrophysics team up

Esoteric relativistic astrophysics finds a parallel in a centuries-old algebraic problem, says S.Ananthanaryanan.

That little known mathematical results have found application in science is commonplace. Even citadels of ***pure*** mathematics, like number theory have become important in computers and E Commerce. The paper by Dmitry Khavinson (University of South Florida) and Genevra Neumann (University of Northern Iowa) in the proceedings of the American Mathematical Society (Jun-July 2008) presents a surprising passage from mathematics to astrophysics.

Algebraic teaser

The so called Fundamental theorem of Algebra is an assertion of in a book on Algebra, published in 1608, that an equation in some variable may have as many solutions for the value of that variable, as the highest power to which that variable is raised in the equation.

To explain, an equation like : $X^2 - 1 = 0$ (x to the power of 2 minus 1 equals zero) is true for the values '1'and '-1' for 'x'. The equation, $X^2 - 5X + 6 = 0$ (x to the power of two minus five times the value of x plus six equals zero) is true for the values '2' and '3' for x. In both these cases, the highest power of 'x' in the expressions considered is '2', and sure enough, there are two values of 'x' that satisfy the equations.

The proposition, back in 1608 was this was true not just for the power of two, but for equations where the variables occurred to any other power, like 5, 7, 100 or 1000, etc.

Attempts to find a proof for this assertion have led to a range of new results and extension of the assertion to other properties of solutions of expressions involving variables in different powers. In 2001, a group of mathematicians had shown that for a certain type of expression, where the highest power was 'n', the values that could satisfy the equation could be at most '3n – 2'.

When Genevra Neumann was a post-doc student at Kansas, she mentioned this result in a talk, and later, she, along with Khavinson, extended the result to a different kind of expression of a variable in many powers, called a '***rational harmonic function***'. They found, with some surprise, that the result was '5n – 5' against the previous '3n – 2'. Well, this was a result of studies in mathematics and the result did not seem to have particular importance. They published their results, anyway, for the record.

Applies to Astrophysics

The publication was seen by an astrophysicist in the Univ of California at San Diego and he telegraphed Neumann to say that her result was just what had been found in a certain kind of problem in Astrophysics, known as ***the Sun Hong Rhie conjecture.***

A phenomenon that is studied in astrophysics is the effect of a number of opaque bodies lying in the path of the light coming from a distant star. In the normal course, an opaque body in the path of light would just block the light and that is all there is. However, when the distances are very large, so that the size of the bodies is small in comparison, there can be some bending of light waves, something like how ocean waves can go 'around' a rock jutting out of the water, or ripples on a pond can 'pass over' a twig sticking out of the water.

When there are a number of such opaque bodies, the lot may act like a lens and focus the image of the distant star at one or more places and the calculations are complex indeed. Albert Einstein went one further and changed the explanation of this kind of 'lensing' by his theory that the gravitation of massive bodies would bend space and hence the path of light. This is called 'gravitational lensing' and again, the mathematics is frightful.

Rhie had been working on gravitational lensing and had ***conjectured*** that a system of 'n' massive bodies in the path of light could produce at most '5n – 5' images. Rhie had also found a way of constructing the mathematical expression, the 'rational harmonic function', with '5n – 5' points of solution.

Wider contacts

That a result in pure mathematics should correspond exactly to a conjecture made by an astrophysicst, in work involving galaxies and black holes is indeed a coincidence that suggests a great field of research that should enrich both mathematics and astrophysics.

When Khavinson and Neumann heard of Rhie's work, they contacted other mathematicians and astrophysicists and refined their understanding of gravitational lensing and the work of other mathematicians. 'Extremely exciting and stimulating interdisciplinary collaboration, say Khavinson and Neumann, when describing the way their work has crossed borders.