The story of a number

TUCKED AWAY IN THE CONTINUUM OF NUMBERS IS ONE LITTLE FIGURE THAT PEEPS OUT AND LINKS TO ALMOST ALL THE IDEAS OF MATHEMATICS, WRITES **S ANANTHANARAYANAN**

2.71 2.715 2.718 2.82818....e.... 2.7183 2.71 84 2.7186 2.72

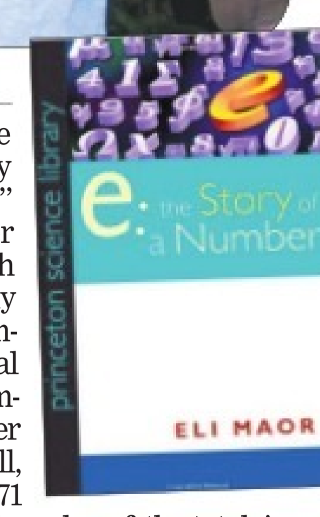
This number, denoted by the letter "e", has the unlikely value, and one that can never be exactly determined, of approximately 2.7182818284… and it turns out to dominate the world of shapes angles, or growth and rates of change, even the progress of numbers themselves. Israel-born Eli Maor, author, historian and teacher of mathematics, in his book, *e: The Story of a Number*, conducts the non-specialist reader through the fascinating course of discovery and the myriad properties of "e" and, along the way, many of the concepts of modern mathematics. In some 200 racy pages, Maor tells the story in simple language, filled with examples and anecdotes to make reading easy as well as rewarding to the newcomer and, equally, to the specialist. A new edition of this book has just been brought out by Princeton University Press.

The number, *e*, arises in mathematics as a result of an infinite process, where a number adds to itself other numbers that grow progressively smaller, in such a way that the total tends to reach, but never gets to some final number. We could illustrate infinite processes by looking at an athlete running a course in a certain time, say a minute. Now to run half the course, he/she takes half a minute, and for the remaining half she takes a quarter of a minute, and for half of what remains, an eighth of a minute, and so on. We can see that here we have a series: ½ + ¼ + 1/8 + 1/16 + 1/32 + 1/64 +…, which will go on to infinite terms as we can always divide any number by two, but the total will not grow beyond one minute, which is the time the athlete takes to run the course. In this case, we also know that the final total will be exactly one minute, because of the way we have stated what was happening.

But there could be a different case where a thing only grows by additions that rapidly reduce in size, to infinite terms, and we need to know how large it gets to be after a certain time. In such a case, the final stage, whatever the size be, may not ever be exactly known, as there is always some tiny addition in the tiny bit of time that is left. An example would be a sum of money that is invested at some rate of interest to double after some given period of time. As the amount doubles in the period, it is clear that it would grow four times in twice the time and only by half in half the time. When a unit sum of money grows by one more unit, we could depict this by the expression: "money grows to (1+1) = 2", in one spell of the given time. The case of two spells would be shown as: "money grows to (1+1)x((1+1) = 2x2 = 4". In the same way, the growth in any number of spells of time, say a number denoted by "n" would be: (1+1) x (1+1) x (1+1) x (1+1)… *n* times. This amounts to (1+1) raised to the power of *n*, and is written as: $(1+1)^n$.

Now let us think of another idea, of the interest during the first spell of time being reckoned not at the end of the period but halfway through and again at the end. In such a case, the amount grows to only (1+1/2) in the first half spell, and it is (1+1/2) that grows by half of that amount in the second half spell. The final amount is thus (1+1/2) x (1+1/2), which can be written as $(1+1/2)^2$, and this comes to 2¼, or 2.25. Now, we could repeat this process by adding interest of one fourth the amount every one fourth of the period, which would make the final amount $(1+1/4)^4$, which works out to 2.3707, a little more than when we split the period into two parts.

Eli Maor and his book



And then by dividing the period into a great many number of parts, say "n" parts, the final number would be $(1+1/n)^n$, which would add a great many but rapidly reducing amounts to the original total amount of "2". As the amounts that are added later in the process are small, indeed, the total crosses 2.71 but does not reach 2.72. The value of the total, in fact, can be expressed as the series: 1+1/1 + 1/2 + 1/6 + 1/24 + 1/120 +… to infinite terms.

This number, which can never, even in principle, be exactly evaluated, was found to have so many remarkable properties that it became a subject of deep study, great advances having been made by the Swiss Leonhard Euler, who first gave the number the name "e". It is, in fact, sometimes referred to as *Euler's number*.

Logs & the calculus

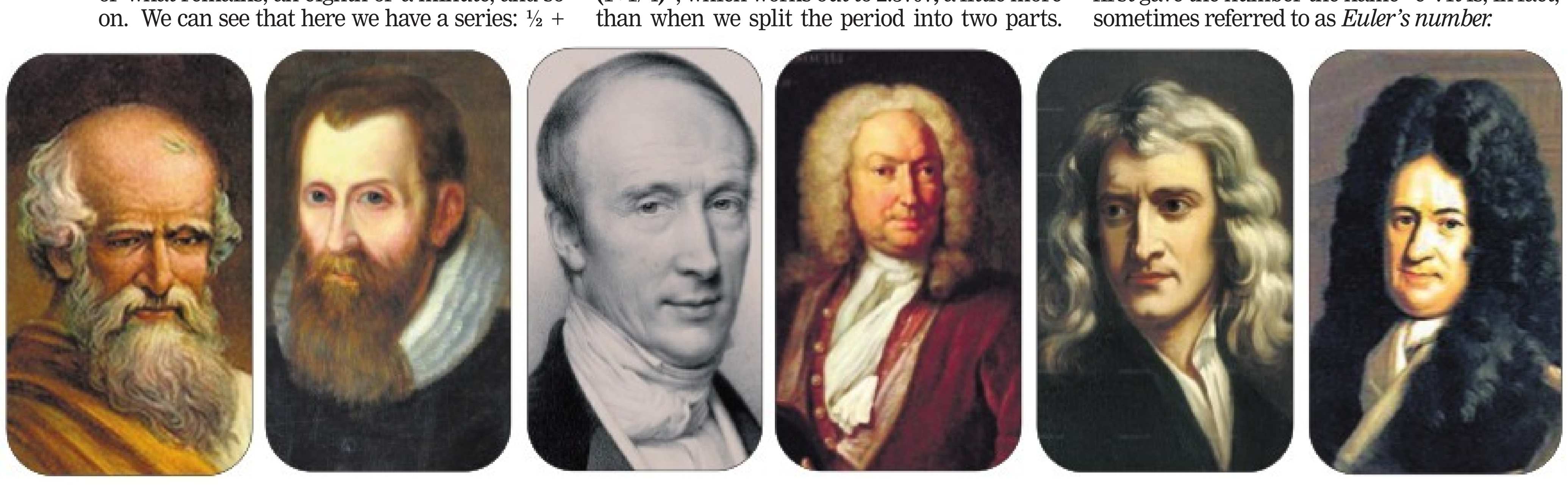
Eli Maor gets the book going with a description, first, of a closely related idea, of logarithms. In the 16th century, Scotsman John Napier noticed that a multiplication, like 4 x 16 = 64, could also be written as $2^2 x 2^4 = 2^6$. When we write the numbers in this way, as powers of the number 2, we notice that the multiplication reduces to simply adding the powers to which the number 2 has been raised, in this case, 2 + 4 = 6. As numbers can also be raised to fractional powers, we could work out a table that shows us the power to which 2 has to be raised to come to any number. With the help of such a table, we could carry out complex multiplications by looking up the power of 2 for each number and then just adding the powers. A reverse table could then tell us the answer from the total that we get. This is the idea of logarithms, or using tables of powers of some base to carry out multiplication with the lesser labour of addition.

Napier's invention soon stabilised with tables based on powers of the number 10, which is easier in the real world than the number 2 as the base, and was surely a great help in the progress in science that followed.

Another development of the period was the discovery of the methods of the *calculus*, independently by Isaac Newton and Gottfried Wilhelm Leibniz. The calculus was first conceived by the Greeks, Archimedes having sown the seeds of making computations by breaking a progression into infinitely small steps. The greater sophistication available in the 17th century allowed more precise definition of relations among variable quantities, like length and breadth or distance and time, and this led the formal and powerful methods of the calculus. One of the concepts in the calculus is the rate at which a value changes and the methods of Newton and Leibniz help work with curved shapes and areas and volumes, forces and motion, growth a nd decay.

Maor's easy narrative painlessly introduces the role of the number *e* in all these fields, like the properties of logarithms to the base *e*, the way the number *e* behaves when it is raised to powers of other numbers and the remarkable property that the value of *e* raised to a power is the same as its own rate of change! As logarithms to the base *e* would involve expressions of *e* raised to different powers, like 2^5 or 10^4, this property of an expression like e^x gains great consequence and is found to play a role in widely separated fields like the values of trigonometric (of the study of triangles) ratios and even the shapes of suspension bridges or the shapes of seashells, galaxies, in art and even musical scales. Even the distribution of prime numbers, in the sequence of integers, is found to be related to the logarithm, to the base *e*, of large numbers!

While the treatment of the math is captivating, equally so are the stories about mathematicians. The rivalry of Newton and Leibniz over credit for discovering the calculus, an imaginary meeting of Johann Bernoulli and JS Bach, and the references to different contributors to the growth of the subject present mathematics as a pulsating, living thing.



Archimedes (from left), John Napier, Augustine Louis Cauchy, Johann Bernoulli, Isaac Newton and Gottfried Wilhelm Leibniz.

THE WRITER CAN BE CONTACTED AT simplescience@gmail.com

PROTEIN ASSEMBLIES

THE ELECTRON TRANSPORT SYSTEM CONSISTS OF FIVE DIFFERENT KINDS OF CARRIERS, SAYS **TAPAN KUMAR MAITRA**

The carriers that make up the electron transport system include flavoproteins, iron-sulphur proteins, cytochomes, copper-containing cytochromes, and a quinone called coenzyme Q. Except the last one, all carriers are proteins with specific prosthetic groups, which are capable of being reversibly oxidised and reduced. Almost all the events of electron transport occur within membranes, so it is not surprising that most of these carriers are hydrophobic molecules. In fact, most of these intermediates occur in the membrane as parts of large assemblies of proteins called respiratory complexes.

Several membrane-bound flavoproteins participate in electron transport, using either Flavin Adenine Dinucleotide or Flavin Mononucleotide as the prosthetic group. FMN is essentially the flavin-containing half of the FAD molecule. An example of a flavoprotein is NADH dehydrogenase, which is part of the protein complex that accepts pairs of electrons from NADH. Another one is the enzyme succinate dehydrogenase, which has FAD as its prosthetic group and is part of the membrane-bound respiratory complex that accepts pairs of electrons from succinate via FAD.

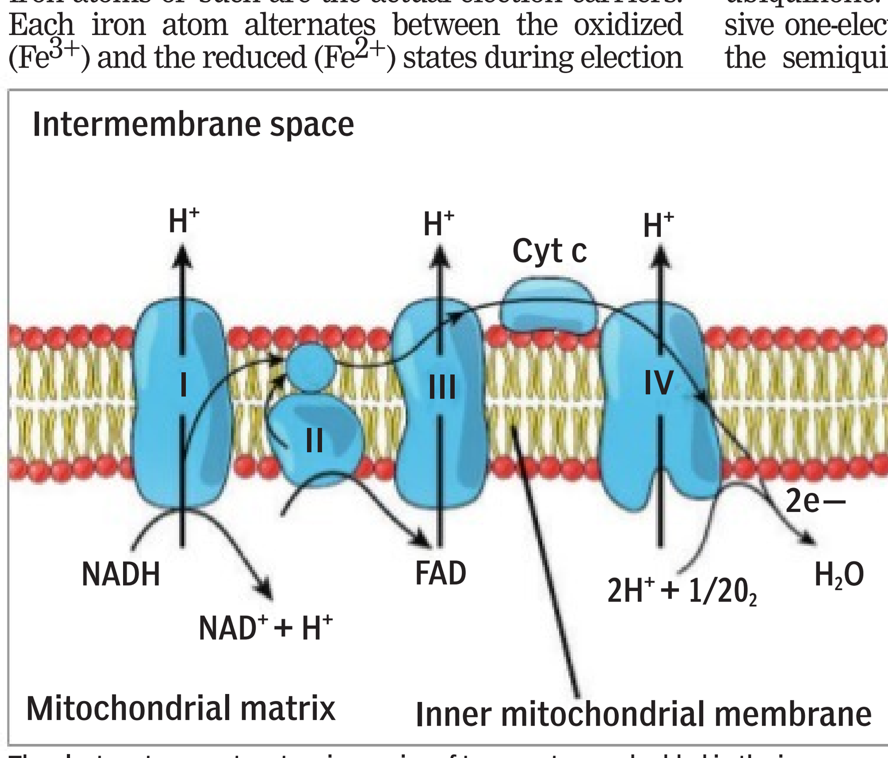
On the other hand, iron-sulphur proteins, also called nonheme iron proteins, are a family of proteins, each with an iron-sulfur (Fe-S) centre, complexed with cysteine groups of the protein. At least a dozen different Fe-S centres are known to be involved in the mitochondrial transport system and the iron atoms of such are the actual election carriers. Each iron atom alternates between the oxidized (Fe^{3+}) and the reduced (Fe^{2+}) states during election transport, which, in this case, involves only one electron at a time. Moreover, iron-sulphur proteins do not pick up and release protons as they cycle between the oxidised and reduced states.

Like the iron-sulphur proteins, cytochromes also contain iron, but as part of a porphyrin prosthetic group called *heme*, which is a component of haemoglobin. There are at least five different kinds of cytochromes in the electron transport system, designated as cytochromes b, c, Cj, a, and a_3. The iron atom of the *heme* prosthetic group, like that of the iron-sulphur centre serves as the electron carrier for the cytochromes. Thus, cytochromes are also one-electron carriers that do not transfer protons. While cytochromes b, c_x, a, and a_3 are integral membrane proteins, c is a peripheral membrane protein that is loosely associated with the outer surface of the membrane. Moreover, cytochrome c is not a part of a large complex and can therefore diffuse much more rapidly, a key property in its role in transferring electrons between protein complexes.

In addition to their iron atoms, cytochromes a, and a_3 also contain a single copper atom bound to the *heme* group of the cytochrome, where it associates with an iron atom to form a bimetallic iron-copper (Fe-Cu) centre. Like iron atoms, copper ions can be reversibly converted from the oxidised (Cu^{2+}) to the reduced (Cu^{+}) form by accepting or donating single electrons. The iron-copper centre plays a critical role in keeping a 0_2 molecule bound to the cytochrome oxidase complex until it has picked up the requisite four electrons and protons, at which point the oxygen atoms are released as two molecules of water.

The only non-protein component of the ETS, coenzyme Q, is a quinone. Because of its ubiquitous occurrence in nature, coenzyme Q is also known as ubiquinone. The reversible reduction, in two successive one-electron steps, from the quinone (CoQ) via the semiquinone (CoQH) to the dihydroquinone, forms $CoQH_2$. Unlike most of the proteins of the ETS, coenzyme Q is not part of a respiratory complex, but instead resides in the non-polar interior of the inner mitochondrial membrane (or of the plasma membrane, in the case of prokaryotes).

It must be noted that coenzyme Q accepts not only electrons but also protons when it is reduced and that it releases both electrons and protons when it is oxidised. This property is vital to the role of coenzyme Q in the active transport, or pumping, of protons across the inner mitochondrial membrane. When CoQ is reduced to $CoQH_2$, it always accepts protons from one side of the membrane then diffuses across the membrane to the outer surface, where it is oxidized to CoQ, with the protons ejected to the other side of the membrane. This is thought to be one of the mechanisms whereby mitochondria, chloroplasts, and prokaryotes establish and maintain the electrochemical proton gradients that are used to store the energy for electron transport.



The electron transport system is a series of transporters embedded in the inner mitochondrial membrane that shuttles electrons from NADH to molecular oxygen. In the process, protons are pumped from the mitochondrial matrix to the intermembrane space, and oxygen is reduced to form water.

THE WRITER IS ASSOCIATE PROFESSOR, HEAD, DEPARTMENT OF BOTANY, ANANDA MOHAN COLLEGE, KOLKATA, AND ALSO FELLOW, BOTANICAL SOCIETY OF BENGAL, AND CAN BE CONTACTED AT tapanmaitra59@yahoo.co.in

Resurrecting Selinunte

THE SITE OF AN ANCIENT MASSACRE IS YIELDING THE SECRETS OF A LOST GREEK CITY, WRITES **DAVID KEYS**

One of the ancient world's greatest tragedies, frozen in time for almost 2500 years, is at last yielding up its long-lost secrets. Archaeologists are gradually unearthing an ancient Greek city — Selinunte in Sicily — whose inhabitants were slaughtered or enslaved by North African invaders in the late fifth century BC. Like the ancient Greek Pompeii, the whole city remained at least partially intact, despite the tragic loss of most of its inhabitants.

At Pompeii all the houses and other buildings were interred almost instantaneously under volcanic ash — but at Selinunte they were buried more gradually by hundreds of thousands of tons of earth and windblown sand. Archaeological excavations are now revealing how the exact moment that Selinunte ceased to exist as a major living city was preserved in graphic detail.

Around 15 per cent of the 250-acre city has, to this day, survived above ground.

Buried under a collapsed roof in a building burnt by the invaders, the archaeologists have even found the half-eaten remains of meals abandoned by the townsfolk as catastrophe engulfed them. Scientists are now analysing visible food residues inside half a dozen bowls left around a hearth in that building. What's more, they have also found dozens of unfired ceramic products — pots and tiles — abandoned by terrified local workers before they'd had a chance to put them in their kilns.

Over the past 15 years, using geophysical techniques and sometimes excavation, the archaeological investigation has so far identified all 2,500 of the long-abandoned city's houses, all its streets, its harbour and its once-flourishing industrial zone. It's the first time archaeologists have been able to produce a detailed comprehensive plan of what a classical Greek city looked like. Previously, they had only been able to gain a relatively fragmentary appreciation of how such cities looked and functioned.

The new knowledge from Selinunte has begun to transform scholars' understanding of some of the key demographic and economic realities of the ancient world as a whole. "Selinunte is the only classical Greek city where the entire metropolis is still preserved, mainly buried under sand and earth. It therefore gives us a unique opportunity to discover how an ancient Greek city functioned," said Professor Martin Bentz of the University of Bonn, director of the major current excavation at Selinunte.

Eighty kilns have so far been identified — including dozens of very large circular ones (used to produce thousands of roof tiles and large ceramic food transport containers) and a dozen large rectangular ones dedicated to producing giant ceramic food storage containers and ceramic coffins! Other smaller kilns were used to make fine tableware, loom weights — and small statues of gods and goddesses. The potters even had their own religious chapel — equipped with altars dedicated to a special working class deity, Athena Ergane (Athena of the Workers) as well as to Artemis (Goddess of Hunting and of Childbirth), Demeter (Goddess of Fertility and of the Harvest) and the king of the gods, Zeus himself.

The archaeology of Selinunte is unique, mainly because the entire city simply ceased to exist as a major population centre in less than a day — as Carthaginian troops (from what is now modern Tunisia) punctured the defences and butchered 16,000 of the Greek inhabitants and soldiers who had been trying to defend it. Some 5,000 more men were taken as slaves, as were many thousands of women and children. Literally from one day to the next, the once bustling city became a ghost town.

Of the tens of thousands of ordinary people who lived there during the 219 years of its existence, only a dozen names have been recovered by the archaeologists — names scratched on the bottoms of drinking cups and jugs found in houses facing the city's great market place.

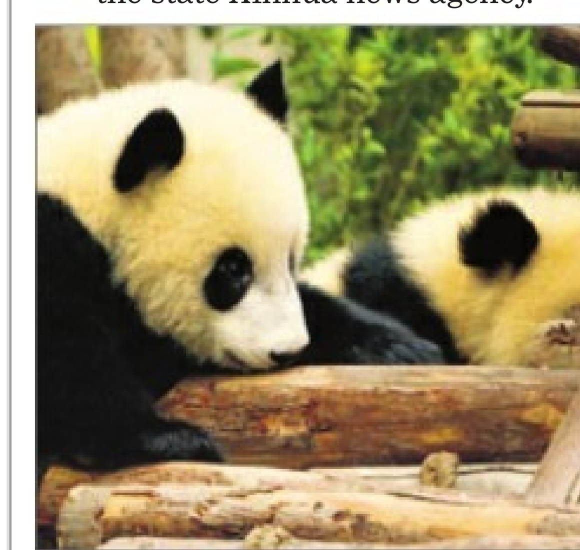
Around 15 per cent of the 250-acre city — mostly its temples and its acropolis — has, to this day, survived above ground. Its jumbled ruins were regarded by participants in the Grand Tour as particularly picturesque and alluring, not just because of its tragic ancient history — but also because the surviving temples had been toppled by a massive earthquake more than 500 years ago. Using their original columns and building materials, two of the temples were re-erected in the mid-20th century and have become major tourist attractions. Selinunte is now the largest archaeological park in Europe.

THE INDEPENDENT

PLUS POINTS

'Panda translator'

During a five-year study of panda "language" at a conservation centre in China's southwestern Sichuan province scientists have found that giant pandas communicate using specific sounds to indicate when they are hungry or unhappy, according to the state Xinhua news agency.



Researchers found that when attracting a mate, males "baa" like sheep and females respond with chirping sounds if they are interested. They also make a "wow-wow" sound when they are unhappy and baby pandas say "gee-gee" to tell their mothers they are hungry.

Zhang Hemin, head of the China Conservation and Research Centre for the Giant Panda, which ran the study, said, "Trust me — our researchers were so confused when we began the project, they wondered if they were studying a panda, a bird, a dog, or a sheep." He said they recorded the animals when they were eating, fighting and nursing young to study how they communicated.

The scientists now plan to use the information to better understand how to protect the critically endangered species in the wild. They say they want to develop a "panda translator" using voice-recognition technology.

Giant pandas are critically endangered with only 1,864 believed to still be living in the wild. Despite a slight recovery in their population reported earlier this year, pandas are still under threat from their well documented fertility problems and the destruction of their habitat.

CAROLINE MORTIMER/THE INDEPENDENT

The Inner Universe

You enter through a dark corridor studded with points of light — green, red, and blue coming from all directions. The lights represent not stellar glimmers but microbial cells. Mirrors on the walls amplify the spots of brightness and you see yourself among them. These aren't just any microbes; they're the trillions of viruses, fungi, bacteria, and protozoans living in and on your body.

"The Secret World Inside You", which opened at the American

Museum of Natural History on 7 November, takes visitors on a tour of the human microbiome — a collection of organisms that weigh two-three pounds, or about the same weight as the brain. And like the brain, our commensal microbes influence the way the body works, from digestion to cognition and behaviour. "Each of us is a complex ecosystem made up of hundreds of species, one of which is human and the rest microbial," show co-curator Robert DeSalle, a scientist in the AMNH's division of invertebrate zoology and the Sackler Institute for Comparative Genomics, said in a statement.

The exhibit includes hands-on activities such as tabletop, arcade-style games where you "shoot" different foods, antibiotics and even probiotics and fecal transplants at your gut microbes and see what happens. The game makes the point that while some microbes can cause harm, so can antibiotics. Other microbes, in contrast, are beneficial.

This is relatively new knowledge. For nearly a century, said DeSalle, the scientific and medical view was that microbes caused disease.

"Then all of a sudden — five years ago, 10 years ago — we started to realise that those things that make us sick are just a tiny part of the things that live in us and on us, and these things that live in us and on us have an importance in our health, also."

ASHLEY P TAYLOR/THE SCIENTIST