'abc' of math in prime time

THE WORLD OF MATHEMATICS IS IN A SPIN WHILE GRAPPLING WITH WHAT MAY BE A BREAKTHROUGH IN THE SCIENCE, WRITES **S ANANTHANARAYANAN**

Shinichi Mochizuki

Mathematician Shinichi Mochizuki of Kyoto University in Japan has published a 500-page document as proof of a math problem of unsurpassed complexity known by the deluding name of the *abc conjecture*, and groups of the world's leading mathematicians are going into huddles to see if it is really so. One group met at Oxford earlier this month and another is currently meeting at Utrecht in the Netherlands.

The *abc conjecture* is so named because it concerns groups of three integers, or whole numbers, referred to simply as a, b, and c, and it deals with fundamental properties of numbers and how prime numbers enter while examining relationships among other numbers. Certain problems that call for solutions in the form of integers are a class of study by themselves and include the legendary *Fermat's last theorem*. If the *abc conjecture* is proved, it would affect a wide area and answer many longstanding questions. "It will be one of the most astounding achievements of mathematics of the 21st century," Dorian Goldfeld, a mathematician at Columbia University in New York, is reported as having said.

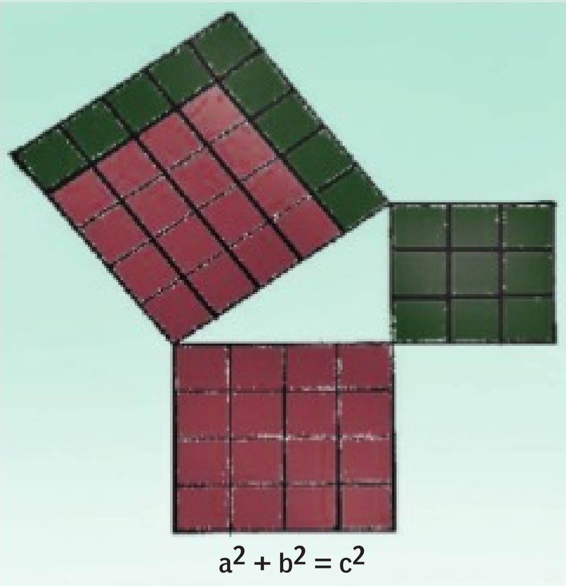
Quite apart from the use of numbers for counting, measurement and describing shapes, forces, time and speed, etc, in the different branches of mathematics the study of numbers themselves has held a fascination all its own and many remarkable results have proved useful in manipulating the physical world. Numbers themselves have no existence, apart from being a quality that is shared by "equal numbers" of groups of things, but with the idea of numbers have grown the concepts of fractions, decimals, infinite, never repeating decimals, binary arithmetic and the science of algebra. And number theory consists of abstractions that examine the relationships between numbers, either integers or the numbers that arise with the use of integers.

Fermat's last theorem is a relationship of numbers, always verified, but which could not be analytically proved since 1647, till it fell to Andrew Wiles of the University of Oxford in 1994. The theorem states that there can be no three integers, say a, b and c, such that $a^n + b^n = c^n$, for a value of n greater than 2. This is the same as saying we can have a + b = c, which is just the sum of any two numbers, a and b, or we can have $a^2 + b^2 = c^2$, but there are no values of a, b and c which satisfy $a^3 + b^3 = c^3$, or any higher powers of the numbers. We can see that here we are dealing with single equations with three variables, a, b and c, and there can hence be several sets of acceptable values, or solutions, and some of them are all in integers. The fact that there is a space of solutions that is speckled with integer solutions is fascinating, and then, to say that there can be no integral solutions for higher powers points to depths in the nature of numbers that mathematicians sought in vain for three and a half centuries.

The *abc conjecture* goes one further and encloses within itself many other mysteries in the ways of numbers. The conjecture, which was proposed independently by the French, Joseph Oesterlé, and Swiss, David Masser, in 1988/1985, is about three numbers that are *relatively prime*, that is, they do not share any prime factors. For example, the numbers 15 and 16 are relatively prime, because the prime factors of 15 are 3 and 5, while 16 has only one prime factor (other than 1), which is 2. This is not true of 18, whose prime factors are 2 and 3, the factor 3 being common with 15. Now the conjecture says that if there are three such numbers, a, b and c, so that a + b = c, then the product of the separate prime factors of a, b and c is generally greater than the number c, and there is a limit to the number of exceptions there can be.

In the case of *Fermat's theorem* and the power 2, we can see that this is a case of a right angled triangle with sides of length 3, 4 and 5, for instance.

We can see that this can be proved graphically by showing that the area of the square on the longest side is equal to the areas of the squares on the other two sides. What *Fermat's theorem* states is that such a construction becomes impossible if we consider the cubes, or any further powers, on the sides of triangles. We can appreciate that the theorem is treating a relationship of multidimensional spaces and numbers.



The square on the longest side exactly contains the squares on the other two sides

The *abc conjecture* deals with a similar relationship and includes prime numbers, of which Euclid proved over 2,000 years ago that there are an infinite number dotting the endless line of integers. There is as yet no formula to say which numbers are primes, but it has been shown that the larger the number, the less likely it is to be a prime. There are also unproved theorems, like the one that says that any number can be expressed as the sum of a pair of primes or that there is an infinite number of pairs of primes that differ by the number 2.

While the conjecture was proposed in 1985, it was in 2012 that Shinichi Mochizuki posted a series of four papers, 500 pages in all, of exceedingly abstruse mathematics purporting to be proof. Mochizuki did not publish the proof in any journal but only on his own website, and even since has not come out to explain his work and resisted interviews. He also does not travel outside Japan and it was only earlier this month that he appeared on Skype at the conference in the University of Oxford. Apart from being voluminous, the proof is said to introduce completely new ideas and an approach and may not have been read and understood by more than a handful of people.

An early presentation at the Oxford meeting is said to have connected the proof with ideas of topology, a branch of mathematics that deals with the relationship of entities whose containers are stretched or twisted. The remaining parts of the meeting, however, are said to have made little progress.

But Mochizuki has a reputation as a creative and responsible mathematician and the community is striving hard to makes sense of the work and, if possible, to put it in a more accessible way. "The exciting aspect is not just that the conjecture may have now been solved, but that the techniques and insights he (Mochizuki) must have had to introduce should be very powerful tools for solving future problems in number theory," Brian Conrad of Stanford University is reported to have said.

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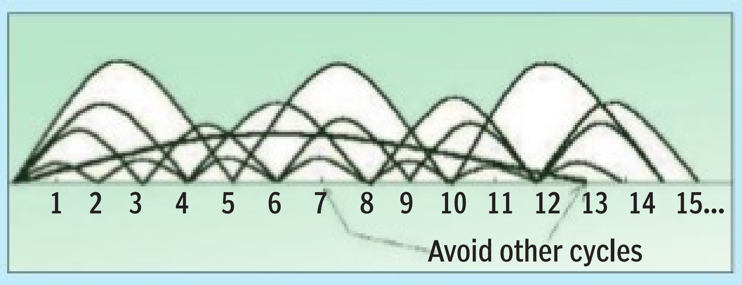
Prime numbers and IT

A form of encoding messages to ensure both authenticity and secrecy is with the help of very large prime numbers. The method works because it is practically impossible to find the factors of a very large number generated from a pair of large prime numbers.

Here, a very large number generated from a secret, large prime number is published as a person's "public key". Messages are then encrypted based on the secret prime number, known as the private key, in a way that the messages can be opened only with the use of the sender's published, public key. If a message opens, it hence proves that it actually came from the purported sender.

Secret messages can similarly be encrypted using an intended recipient's public key. These messages can then be opened only with the help of the private key of the receiver, and this ensures that others cannot read a message that falls into their hands.

Prime numbers in nature



Some insects that stay hidden and safe in burrows till they emerge from time to time to breed, make use of prime numbers to avoid predators. If they were to emerge at the same time every year, the predators would be there right when they appear and there may not be much breeding that season. Even if they were to delay their emergence to be once every say, eight years, they would still be at risk of predators that appeared every eight, four, or two years. A species that breeds every six years would similarly risk predators that come every six, three or two years. But if the species bred every seven years, the risk would be only once every seven years, as seven has no divisors.

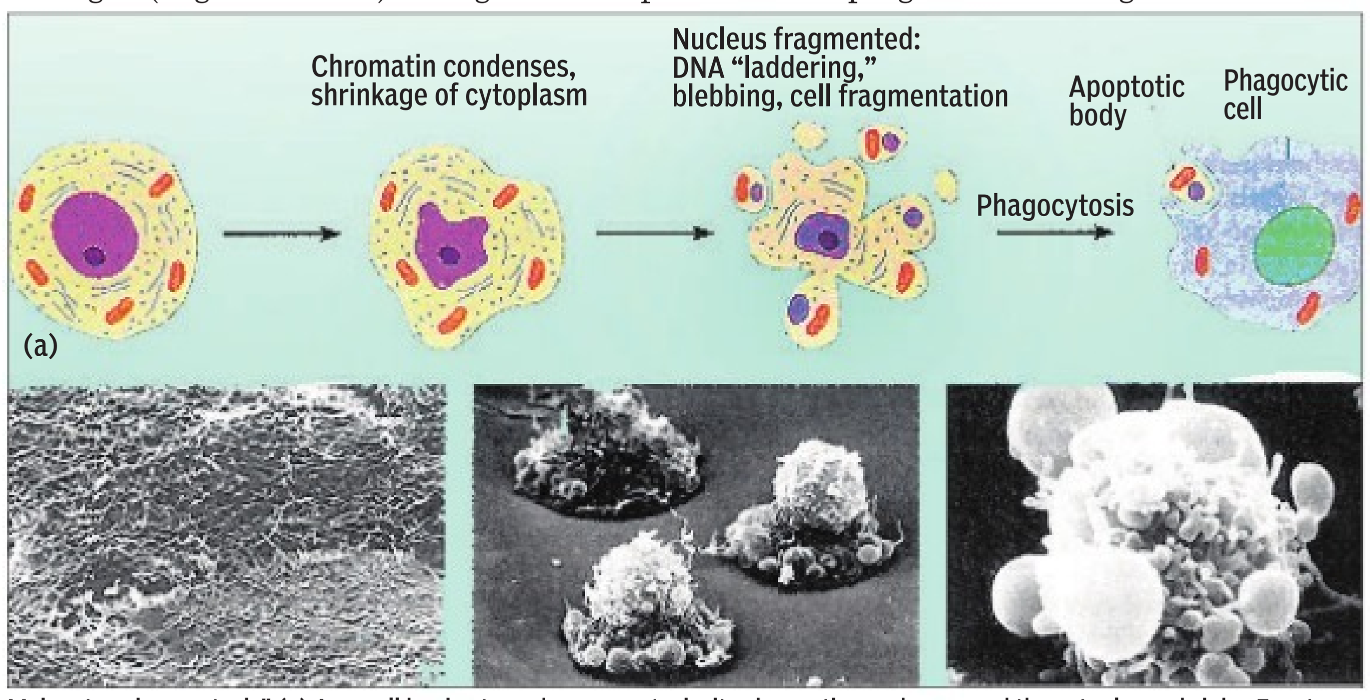
Varieties of Magicuda, a celebrated "periodic" strain of grasshopper, stay underground for either 13 or 17 years before they merge for a brief six weeks of feeding and breeding. The prime number of years of hibernation seems to be their secret of survival.

Mario Marcus and others at the Max Planck Institute of Molecular Physiology in Dortmund developed a mathematical model of competing species with different reproductive efficiency, feeding habits and cyclicity. Computer simulation to discover the most efficient cycles was found to work as a prime number generator!

HOW CELLS PERISH

A KIND OF PROGRAMMED DEATH IS A KEY EVENT IN MANY BIOLOGICAL PROCESSES, WRITES **TAPAN KUMAR MAITRA**

Signal transduction is not only important during hormonal regulation but also in growth factor signaling. The primary reason for that being cell signalling also regulates a kind of programmed death or apoptosis, which is a key event in many biological processes. In embryos, apoptosis occurs in a variety of circumstances and examples include removal of the webbing between the digits (fingers and toes) during the development of hands and feet, the resorption of the tail of tadpoles when they undergo metamorphosis and the "pruning" of neurons that occurs in human infants during the first few months of life as connections mature within the developing brain.



Major steps in apoptosis" (a) As a cell begins to undergo apoptosis, its chromatin condenses and the cytoplasm shrinks. Eventually the nucleus becomes fragmented, its DNA is digested at regular intervals ("laddering"), the cytoplasm becomes fragmented, and the cell extends numerous blebs. Ultimately the remnants of the dead cell (apoptotic bodies) are ingested by phagocytic cells, (b-d) SEMs of epithelial cells undergoing apoptosis. (b) Epithelial cells in contact with one another in culture form flat sheets. (c) As apoptosis ensues, the cells round up, withdraw their connections with one another. (d) A single dead cell with many apoptotic bodies.

In adult humans, apoptosis occurs continually — when cells become infected by pathogens or when white blood cells reach the end of their lifespan, they are eliminated through apoptosis. As a result, millions of cells die every minute in the human body. When cells that should die via apoptosis do not, the consequences can be dire. We now know that mutations in some of the proteins that participate in apoptosis can lead to cancer — melanoma frequently results from a mutation in Apaf-1, a protein active during the process.

Apoptosis is very different from another type of cell death, known as necrosis, which sometimes follows massive tissue injury. Whereas necrosis involves the swelling and rupture of the injured cells, apoptosis involves a specific series of events that leads to the dismantling of its internal contents. During the early phases of apoptosis, the cell's DNA segregates near the periphery of the nucleus and the volume of the cytoplasm decreases. Next, it begins to produce small bubble-like cytoplasmic extensions ("blebs") while the nucleus and organelles begin to fragment.

The cell's DNA is cleaved by an apoptosis-specific DNA endonuclease, or DNase (an enzyme that digests DNA), at regular intervals. As a result, the DNA fragments, which are multiples of 200 base pairs in length, form a diagnostic "ladder" of fragments. Eventually the cell is dismantled into small pieces called apoptotic bodies. Ultimately the remnants of the affected cell are engulfed by others nearby (typically macrophages) via phagocytosis. The macrophages act as scavengers to remove the resulting cellular debris.

That cells have a "death programme" was first conclusively demonstrated in the nematode, "Caenorhabditis ele-gans", where key genes that control apoptosis were first identified. Subsequent research showed that many other organisms, including mammals, use similar proteins during apoptosis. As a result of such studies, many of the molecular events that lead to the process are now known.

A key event in apoptosis is the activation of a series of enzymes called caspases. (Caspases get their name because they contain a cysteine at their active site, and they cleave proteins at sites that contain an aspartic acid residue followed by four amino acids that are specific to each caspase).

Caspases are produced as inactive precursors known as procaspases, which are subsequently cleaved to create active enzymes, often by others of the ilk, in a proteolytic cascade. Once they are activated, caspases cleave other proteins, resulting in efficient and precise killing of the cell in which they are activated. The apoptosis-specific DNase is a good example — it is bound to an inhibitory protein that is cleaved by a caspase.

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Why old birds never grey

SCIENTISTS HAVE DISCOVERED HOW THEY ARE ABLE TO PRODUCE INTENSELY-COLOURED PLUMAGE BY MANIPULATING THE WAY LIGHT IS REFLECTED FROM THEIR FEATHERS, WRITES **STEVE CONNOR**

Scientists have discovered why old birds never go grey with age, why green tree frogs turn blue when dead and why a red sweater might in future be washed with white shirts without turning them pink. It all comes down to the novel way that many birds are able to produce intensely-coloured plumage by manipulating the way light is reflected from their feathers.

A study of a single barb in a jay's wing feather has revealed that it contains a network of sub-microscopic holes within a spongy matrix that accounts for the overall hue of the bird's plumage, scientists said. The research has shown that the colour of a feather barb can turn from white to blue purely as a result of the change in the size of the holes within the spongy filling — the first time that bird colouration has been explained in such minute detail.

The study can explain why the brightly coloured feathers of many birds do not fade in sunlight or go grey with age, unlike human hair that relies on the continuous production of the dark pigment *melanin* as each hair grows from its follicle. The results of the study might also lead to the invention of new types of "structural" colours, which are not based on pigments, for commercial paints and dyes that never fade in sunlight or run in the wash, the researchers said.

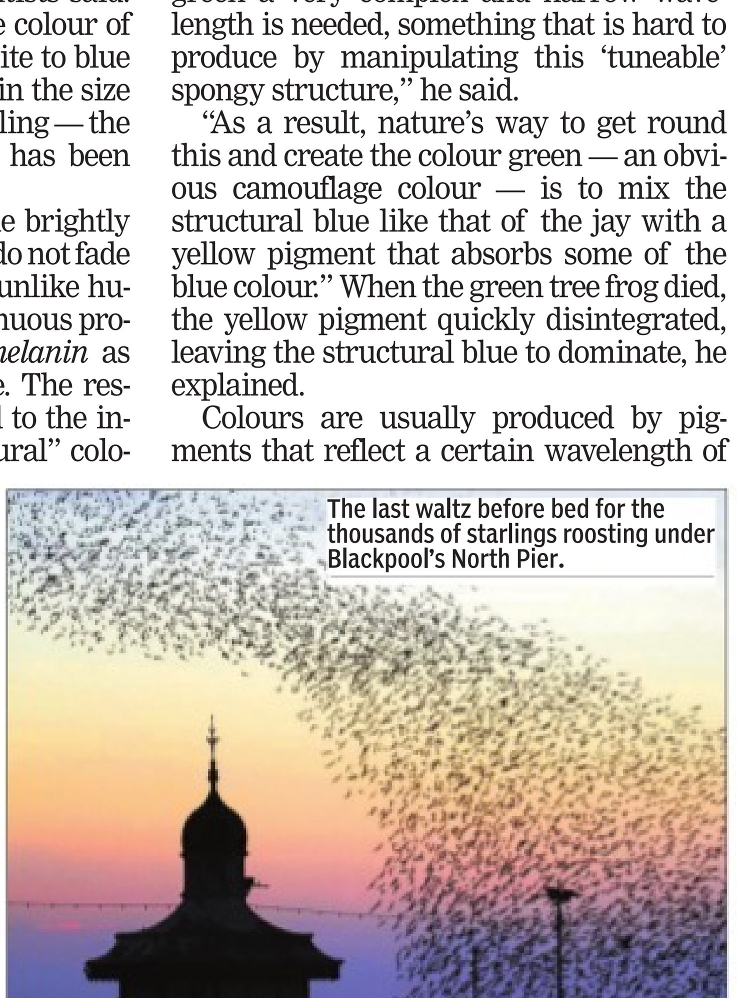
"This discovery means that in the future we could create long-lasting coloured coatings and materials synthetically," said Andrew Parnell of the University of Sheffield, lead author of the study published in the journal *Scientific Reports*.

The researchers used a powerful X-ray instrument called the European Synchrotron Radiation Facility in Grenoble to build up detailed images of the spongy nanostructure within the jay's feather barb, down to a scale of a billionth of a metre. They found that the size of the holes within the spongy layer, which are under genetic control, determined which wavelengths of light — and hence colour — were reflected and so seen by other birds or humans.

"We have discovered it is the way in which it is formed and the control of this evolving nanostructure — by adjusting the size and density of the holes in the spongy-like structure — that determines what colour is reflected," Dr Parnell said. "Current technology cannot make colour with this level of control and precision — we still use dyes and pigments. Now we've learnt how nature accomplishes it, we can start to develop new materials such as clothes or paints using these nanostructuring approaches. It would potentially mean that if we created a red jumper using this method it would retain its colour and never fade in the wash."

The finding could also explain the unusual colouration of other animals, such as green tree frogs that mysteriously turn blue after death, said Adam Washington of Sheffield University, a co-author of the study. "This is because to create the colour green a very complex and narrow wavelength is needed, something that is hard to produce by manipulating this 'tuneable' spongy structure," he said.

"As a result, nature's way to get round this and create the colour green — an obvious camouflage colour — is to mix the structural blue like that of the jay with a yellow pigment that absorbs some of the blue colour." When the green tree frog died, the yellow pigment quickly disintegrated, leaving the structural blue to dominate, he explained.



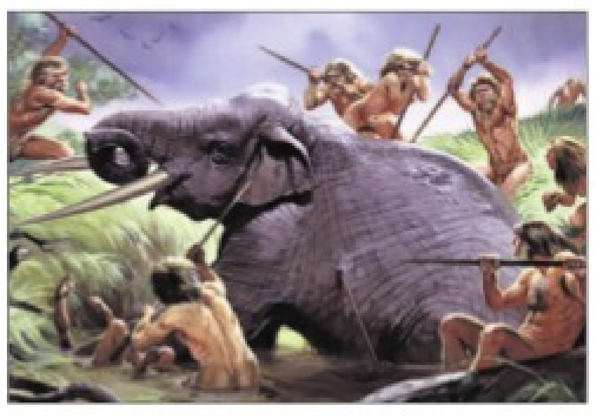
The last waltz before bed for the thousands of starlings roosting under Blackpool's North Pier.

Colours are usually produced by pigments that reflect a certain wavelength of light to produce a particular colour of the visible spectrum. Other visible wavelengths are absorbed by the pigment, making them invisible. However, it has been known since the days of Isaac Newton and Robert Hooke in the 17th century that there is another way of producing colours using microscopic layers or surfaces that reflect light waves in such a way that they interfere with one another.

This so-called structural colouration is known to play a role in many kinds of vivid displays in nature, such as the iridescent wings of beetles to the brightly coloured feathers of peacocks and kingfishers.

THE INDEPENDENT

PLUS POINTS



The more northerly groups of "Homo heidelbergensis" developed into Neanderthals, while the more southerly evolved to become "Homo sapiens".

What our ancestors looked like

Until now, scholars had thought the split that eventually led to the development of *Homo sapiens* (modern humans) and *Homo neanderthalensis* (Neanderthal) had taken place in Africa or Asia some 400,000 years ago. But new research from Cambridge University suggests the split probably occurred at a much greater time depth, around 600,000 to 700,000 years ago.

As well as revealing for the first time what our immediate "pre-split" ancestors looked like, it also therefore shows the unexpectedly vast amount of time it took for that ancestral species to eventually evolve into modern humans in Africa and into Neanderthal humans in West Asia and Europe. The researchers' next step will be to similarly computer-model that common ancestor's lower jaw and chin and then finally use modern forensic techniques to digitally put flesh and skin on the complete skull to produce the first scientifically-deduced "photofit" of what our long-lost ancestors actually looked like.

The new date for the original evolutionary split also suggests the possibility that climate may have played an even greater role in the story than previously thought. In around 700,000 BC much of Africa's climate became more frequently wetter and, as a result, the Saharan region became intermittently much less arid. This is turn almost certainly allowed an African human species called *Homo heidelbergensis* to expand into previously inaccessible territories on that continent.

The Cambridge scientists used so-called digital morphometrics and statistical algorithms to extrapolate forward to and backwards from known morphological data obtained from 15 fossilised early human skulls and skull fragments from both before and after the split. They used the data to plot changes across time of almost 800 different "points" on the "topography" of the evolving prehistoric human face and cranium, including that of *Homo erectus*, the first species of human to migrate out of Africa about 1.9 million years ago.

"We have long known that we share a common ancestor with Neanderthals, the extinct species that were our closest prehistoric relatives. But what that ancient ancestral population looked like has, until now, remained a mystery, as fossils from the period during which the lineage split, are extremely scarce and fragmentary," said Aurélien Mounier, the Cambridge scholar who carried out the modelling work.

DAVID KEYS/THE INDEPENDENT

Tut's wet nurse

Archaeologists studying the tomb of the Egyptian boy-pharaoh Tutankhamun's wet nurse Maya say she may have actually been his half-sister Meritaten. On 20 December, French archaeologist Alain Zivie announced his findings as he unveiled Maya's tomb to journalists ahead of its public opening next month.

French archaeologist Alain Zivie shows journalists the carvings on Maya's tomb.

Dr Zivie discovered her tomb in 1996 at Saqqara, a necropolis about 20 km south of Cairo. She was believed to be the wet nurse of Tutankhamun — whose mummy was famously discovered by British archaeologist Howard Carter in the Valley of Kings in Luxor in 1922. Zivie said his conclusion about Maya's identity was based on carvings of her and Tutankhamun on the walls of her tomb. He told AFP, "The extraordinary thing is that they are very similar. They have the same chin, the eyes, the family traits. The carvings show Maya sitting on the royal throne and he is sitting on her.

Egyptian antiquities minister Mamdouh el-Damaty said the tomb included scenes of Maya nursing her brother.

CAROLINE MORTIMER/THE INDEPENDENT