Babylon scoops Greece in math

Long before Pythagoras and Euclid, the Babylonians had a firm grip on the mathematics of triangles

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Much of mathematics derives from or can be reduced to the properties of triangles. An ancient scrap of a clay tablet, unearthed early in the last century in Iraq, has been found to present the properties of triangles with unprecedented sophistication and accuracy. Interpretation of the markings on the tablet, which has only now been worked out, has been found to displace the Greeks of the sixth century BCE as originators of the field of trigonometry, handing the position over to the Babylonians of a thousand years before.

Daniel F Mansfield and N J Wildberger of the University of New South Wales, Sydney, Australia in their paper in the journal, *Historia Mathematica*, demonstrate that the markings on the tablet represent real mathematical computation of great complexity, rather than an "unwitting creation of an effective trigonometric table", as has been suggested.

The place of ancient Greek geometers as the forerunners of mathematics has been undisputed. While Euclid formalised the study of lines, circles, angles and areas, there is a special place for his predecessor, Archimedes, who made a seminal discovery about the right-angled triangle. This property, that squares drawn on the two shorter sides of a right-angled triangle are equal in area to the square drawn on the longest side, leads to the idea that the ratio of the sides of a right-angled triangle stays unchanged when the triangle grows or shrinks and depends only on the two lesser angles in the triangle.

The fact that this is the relationship among the sides of all triangles of the same shape leads to a way to measure the angles, marked as α and β in the picture, based on the lengths of the sides. The ratio of the shorter sides to the longest, for different angles, α and β, have been worked out and tabulated as the so-called trigonometric tables. These tables, which state the ratios for all possible angles — correct to less or more decimal places — are of great use to scientists and engineers. Given the length of one side of a right-angled triangle, we are able to work out the lengths of the other sides with the help of the other angles and a set of trigonometric tables.

The lengths of the sides of the triangle in the example shown are all whole numbers. Whole numbers, where the sum of the squares of two of the numbers is the square of the third number, like in a right-angled triangle, exists only for certain triads, or groups of three, of whole numbers. These sets of three numbers are known as Pythagorean triples. In such cases, the ratio of the lengths of the sides, and hence a measure of the angles in the triangle, can be worked out with great accuracy. In other cases, lengths need to be measured and the ratios are approximations. In principle, there is a triad of whole numbers that represents the ratio of the sides of any right-angled triangle, but discovering this triad, or the relevant Pythagorean triple, is not a trivial matter.

Other methods of working out these ratios, for pairs of lines that meet at an angle, were developed in later years. The Greek, Hipparchus, of the second century BCE developed a way by using chords of a circle. Madhava of the Kerala School of Astronomy, in the 14th century, developed a way using the sum of many terms of an infinite series of reducing fractions and this was followed later by Leibniz of Germany and others.

The Babylonian artefact, catalogued as Plimpton 322 as it was sourced by the publisher, George Arthur Plimpton, is a clay tablet with 15 rows of sets of three numbers engraved in cuneiform. The notable feature of the engraving is that the three numbers in each set are related to Pythagorean triples. Specifically, the second and third numbers are the shortest and the longest sides of right-angled triangles, while the first number is related to the third side.

The origin of the tablet has been placed at Larsa, an ancient Sumerian city near the Persian Gulf and its antiquity, by comparison with other Old Babylon scripts, has been placed at between 1822 and 1762 BCE. The tablet itself is 12.8 cm by 8.8 cm in size and contains the 15 rows of writing, each with the sets of three numbers and then a serial number.

The numbers are written in the Babylonian sexagesimal number system, or numbers with a base of 60, in place of the base of 10 in our decimal system. There are thus separate symbols for the numbers from 1 to 59 (zero is denoted by a space) and the number 60 would be represented by the symbol for "one", followed by a space, to form "10", or one time the base (60) plus zero units. This system of counting, the paper explains, is richer and enables much higher exactness than the decimal system. The Old Babylon mathematics also looks at triangles from the point of view of the lengths of the three sides, without the notion of the angles, the paper says.

Modern trigonometric tables, which scientists and engineers use, are lists, for a range of angles, of the ratios of the sides of a right-angled triangle where the longest side is one unit long. The Plimpton 322 tablet, on the other hand, the paper says, is a series of sets of three whole numbers, where each set allows the ratios of the trigonometric table to be calculated, for a useful range of slope of triangles, with great accuracy. With this interpretation, the paper says, the Babylonian tablet becomes not only the oldest recorded trigonometric table but also the most accurate one to date, as it is exact, with no approximation!

This function of the Plimpton tablet has been an unanswered question. Ever since the markings on Plimpton 322 were identified as Pythagorean triples, there has been extensive research into how the Babylonians may have worked out the numbers out and why they did it. The University of New South Wales authors review some suggestions, like the tablet being a teacher's aide or a representation of a class of right angled triangles and succeed in showing that these cannot be the case. On the other hand, they do show that the 15 rows provide a mechanism to evaluate dimensions of triangles that correspond to angles from 45^{o} to 59^{o}, a range that would be relevant in normal construction work.

The paper reviews the tradition of Old Babylon mathematics, the geometry devised for civil work, and the Egyptian ratio-based measurements used to describe pyramids. And several other pieces of evidence, of ratios used to represent a slope, lead to a "ratio based framework for the study of triangles", without the use of the angle as a metric, the paper says.

And there are examples, like finding the length of a side of the Great Pyramid of Gaza, given the height and the width of the base. The tablet of Babylon gives a better answer than the procedure that uses the table of Madhava!

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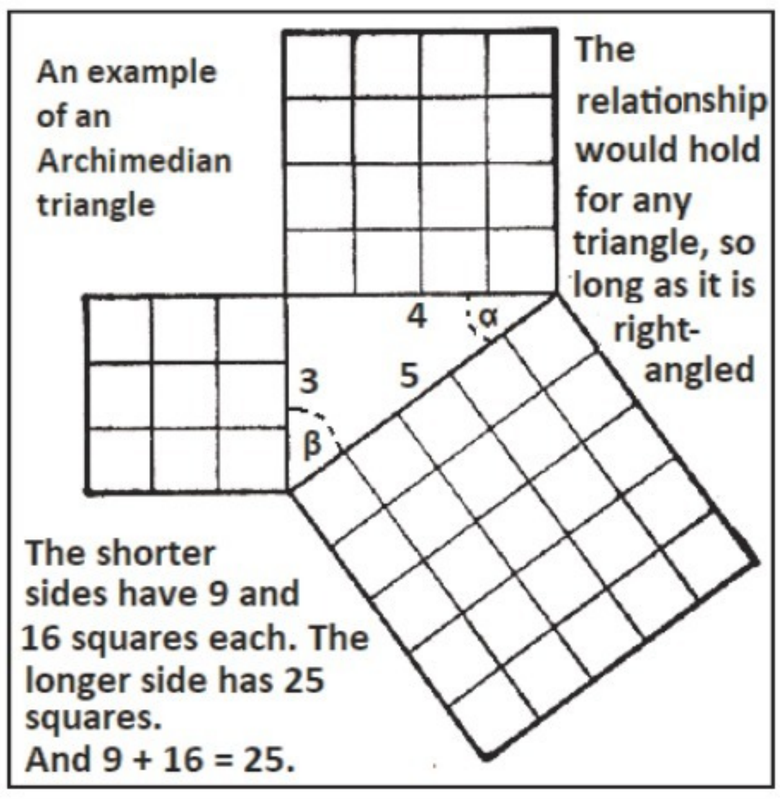


PLUS POINTS

Easily outlast men

Women have greater muscle endurance than men, a study appears to show.

Researchers from the University of British Columbia found women were less tired after natural muscle exercises than men of a similar age and athletic ability. For the study, nine women and eight men were asked to flex their foot against a series of sensors as quickly as they could 200 times. The speed, power and torque — rotational force — of their movements and electrical activity of their muscles was recorded.

The results showed men were faster and more powerful at first, but became more exhausted much faster than the women. Professor Brian Dalton, study author, said, "We've known for some time that women are less fatigable than men during isometric muscle tests — static exercises where joints don't move, such as holding a weight — but we wanted to find out if that's true during more dynamic and practical everyday movements.

"And the answer is pretty definitive — women can outlast men by a wide margin." The researchers measured foot movements because it makes use of calf muscles on the back of the leg, used for everyday actions such as standing or walking.

Although only one muscle group was studied, Dalton said he would expect similar results for others. "We know from previous research that for events like ultra-trail running, males may complete them faster but females are considerably less tired by the end," he explained.

"If ever an ultra-ultra-marathon is developed, women may well dominate in that arena." The study, carried out in collaboration with the University of Guelph and University of Oregon, was published in the journal, *Applied Physiology, Nutrition and Metabolism*.

The independent

Pollution monitor

Delicate moss found on rocks and trees in cities around the world can be used to measure the impact of atmospheric change, and could prove a low-cost way to monitor urban pollution, according to Japanese scientists.

The bio-indicator responds to pollution or drought stress by changing shape, density or disappearing, allowing scientists to calculate atmospheric alterations, said associate professor Yoshitaka Oishi from Fukui Prefectural University. "This method is very cost-effective and important for getting information about atmospheric conditions," Oishi told the Thomson Reuters Foundation by telephone. "Moss is a common plant in all cities, so we can use this method in many countries. They have big potential to be bioindicators," said Oishi, who analysed nearly 50 types of moss for the study. He said humid cities where moss thrive could benefit most.

In a research paper published in the Landscape And Urban Planning journal, Oishi and a colleague described how they studied the effect of nitrogen pollution, air quality and drought stress on moss found over a three sq km area in Hachioji city, in north-western Tokyo. The study showed that severe drought stress tended to occur in areas with high levels of nitrogen pollution, which it said raised concerns over the impact on health and biodiversity.

The World Health Organisation says 88 per cent of city dwellers are exposed to annual pollution levels that exceed its air quality guidelines. South-east Asia and the eastern Mediterranean have the worst air quality.

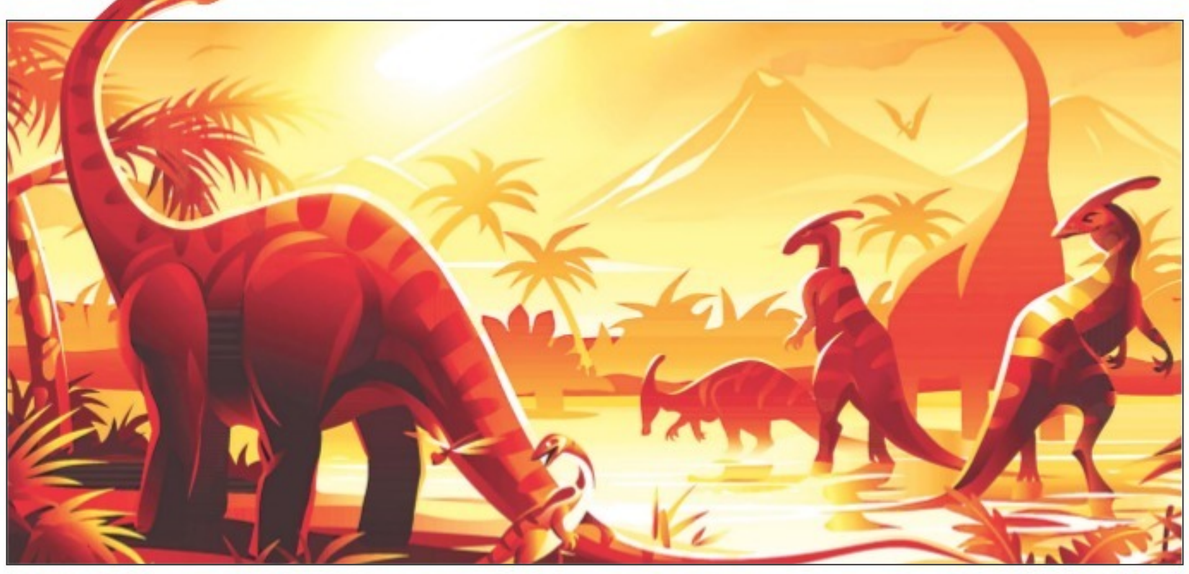
"We believe this method can contribute to the evaluation of atmospheric pollution in other areas," said Oishi.

The straits times/ann

Endless night and prolonged winter

Research into the massive asteroid strike's effects on the climate — when dinosaurs were wiped out 66 million years ago — could help scientists work out what will happen in the event of a nuclear war

IAN JOHNSTON



It is a vision of hell on Earth. The sun disappeared behind a cloud of smoke that encircled the planet, turning day into night and causing temperatures to plummet as fire rained down from above. And if that wasn't bad enough, the massive asteroid that struck the planet 66 million years ago — wiping out the dinosaurs and many other species in the fifth mass extinction of all life — set off earthquakes, giant tsunamis and volcanoes.

Even after the dust cleared nearly two years later, chemical reactions in the upper atmosphere would have wrecked the protective layer of ozone so that the sun's rays would have irradiated anything left alive by the strong ultraviolet light.

It seems extraordinary that life survived such an Armageddon-like event, which was laid bare by researchers who used a sophisticated computer model to work out the effect of the 10km-wide asteroid on the climate. They said their work, supported by Nasa, could help work out how the atmosphere would be affected by the detonation of large nuclear bombs and the chances of the feared "nuclear winter".

But it is also worth considering other scientists' warnings that human activity is ushering in the sixth mass extinction of life on the planet, with the loss of animal life on a par with the astonishing destruction revealed by the new research.One of the researchers, Charles Bardeen, of the US National Centre for Atmospheric Research, said the initial, devastating impact of the asteroid would have been just the start of the Earth's troubles.

"The extinction of many of the large animals on land could have been caused by the immediate aftermath of the impact, but animals that lived in the oceans or those that could burrow underground or slip underwater temporarily could have survived," he said, "Our study picks up the story after the initial effects — after the earthquakes and the tsunamis and the broiling. We wanted to look at the long-term consequences of the amount of soot we think was created and what those consequences might have meant for the animals that were left." Previously scientists have estimated that about 15 million tons of fine soot was created and carried in the wind around the world following the asteroid strike.

The researchers in the new study, described in an article in the journal *Proceedings of the National Academy of Sciences*, found that this would have reduced the amount of light during the day to about the same as the night when there is a full moon. That would have prevented plants from photosynthesising, causing major problems for all life on the planet. Average temperatures would have fallen by about 28 degrees Celsius on land and 11oC over the oceans.

It is estimated that more than three-quarters of all species on Earth, including most of the dinosaurs, became extinct during this time. The asteroid strike, known as the Cretaceous-Paleogene (K-Pg) extinction event, vapourised rock in the air, which then condensed into small particles known as spherules. As these fell to the ground, the friction would have made them so hot that they would have started fires on the ground. Today a thin layer of spherules can be found all over the world.

Photosynthesis would have been impossible for about 18 months, the model predicted. Meanwhile in the upper atmosphere, the soot in the upper stratosphere layer would have absorbed energy from the sun, warming it by more than 200^{o}C. This would have led to the destruction of the ozone layer, which protects life on Earth from harmful levels of radiation, and also enabled vast amounts of water to be stored in the stratosphere. The water vapour would have caused yet more damage to the ozone.

So as the skies cleared of soot, the sun's rays would have irradiated life on the ground but it also began a rapid cooling process. Less soot meant the stratosphere cooled, so the water vapour condensed and washed out some soot as it fell to the ground. That created a feedback loop that removed the soot layer in just a few months. Dormant seeds burst into life, mammals came out of their burrows, the ozone layer slowly built up again and, gradually, life on Earth started to recover.

The researchers cautioned that their model had used one of the Earth today, which is slightly different to the one 66 million years ago, in terms of the gases in the atmosphere and the position of the continents. And Bardeen said the model used also had its limitations. "An asteroid collision is a very large perturbation — not something you would normally see when modelling future climate scenarios," he said, "So the model was not designed to handle this and, as we went along, we had to adjust the model so it could handle some of the event's impacts, such as warming of the stratosphere by over 200^{o}Celsius."

However, he added that their results could help scientists work out what might happen in the event of widespread nuclear war. "The amount of soot created by nuclear warfare would be much less than we saw during the K-Pg extinction," Bardeen said, "But the soot would still alter the climate in similar ways, cooling the surface and heating the upper atmosphere, with potentially devastating effects."

The independent