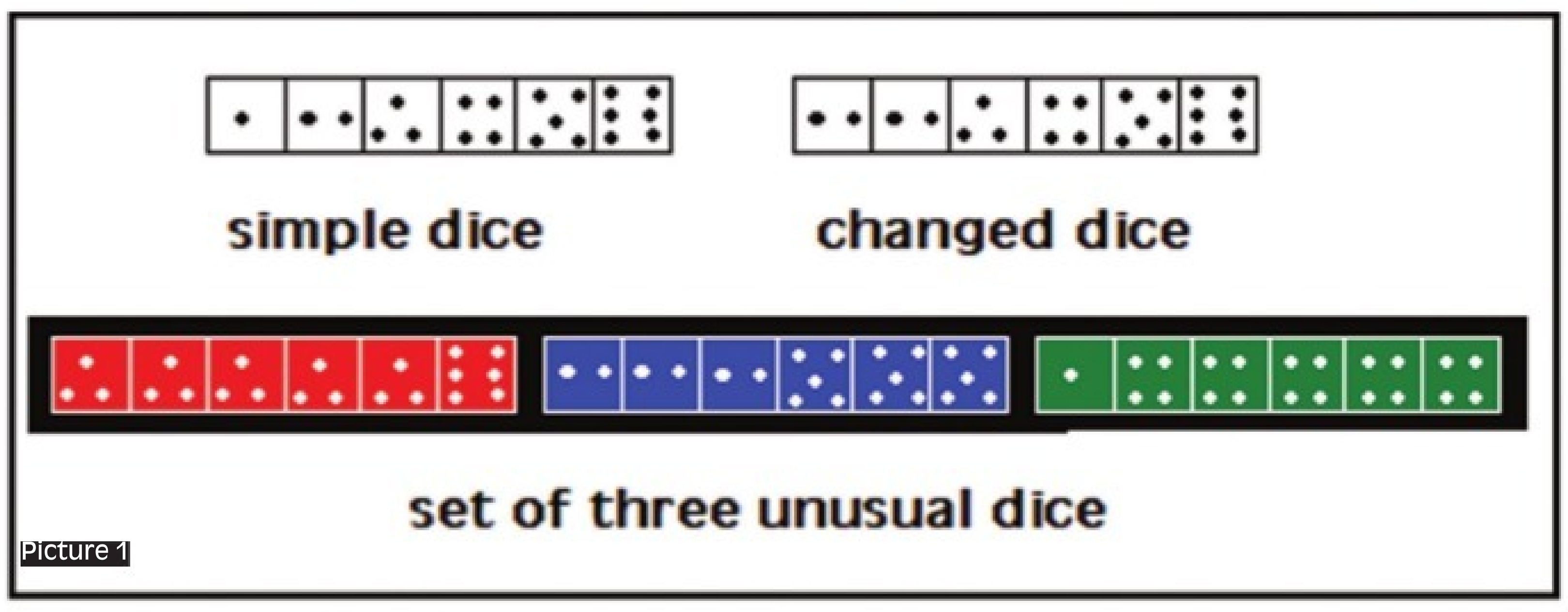
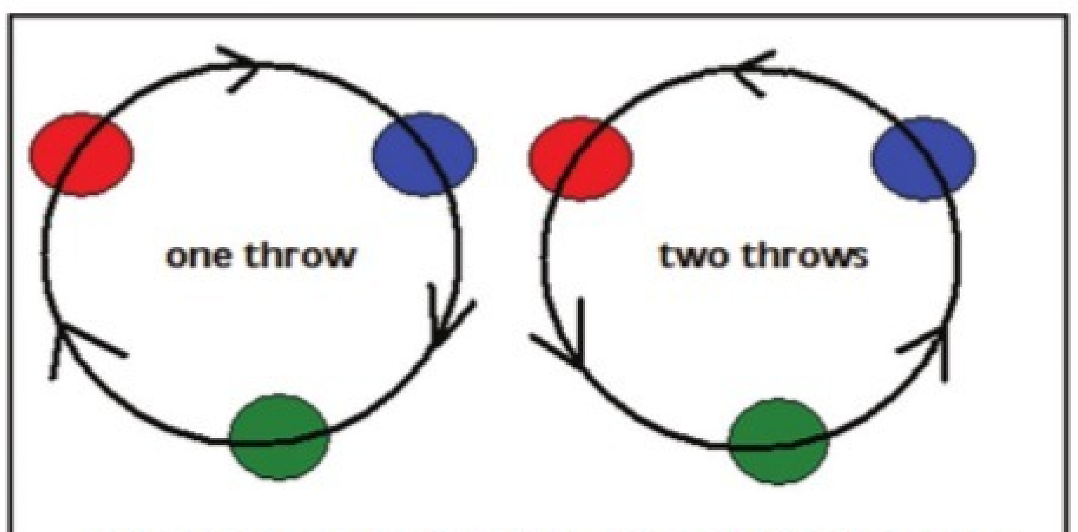
Playing with numbers



Picture 1

A series of selected writing explores the range and oddities of math, as well as its surprising connections with the mind, life and society

S ANANTHANARAYANAN



Picture 2

The Princeton University Press has brought out the ninth volume of a series, since 2010, of selected writing that explores the range and oddities of math, as well as the connections of math with minds, life and society. For the most part, the collection, "Best writing on mathematics" is accessible to persons without special training in math and informative for mathematicians as well.

An instance in the collection is the piece on the chances of winning at a game of dice. As we know, a dice has six sides, marked as shown in the first picture.

And there is an equal chance that either of two players would roll a higher or lower number. But if one of the dice had altered faces, like in picture, which is with two sides marked as a throw of "2" and none of "1". It is easy to see that this dice would win more often than the other one.

The author of the piece now proposes a set of three unusual dice, the third part of the picture. The red dice has five sides marked with "3" and one with "6". The Blue has three sides marked with "2" and three sides with "5". It is easily worked out that the red dice beats the blue dice seven times out of 12 throws. And similarly, if we consider the blue and green dice, we can work it out that the blue dice beats the green dice, which has five sides marked with "4" and one side with "1", the same seven times out of 12.

Now comes the surprising part. When we consider the red and the green dice, it is the green dice that beats the red dice, 25 times out of 36 throws, which is better than seven times out of 12! It turns out to be like a game of rock, paper and scissors! The three dice work in a circle like in the second picture.

And the marvel does not stop here, if the game is played with two throws of the dice in place of one throw, the order of scoring reverses, as shown in the picture.

The article then examines the case of playing with more than two players and more than three dice and the possibility of beating either or both players and successive cases of increasing complexity.

Another article in the book examines the popular club and party game of Bingo, also known as Housie or Tambola. The game is played with cards that have five columns and five rows. Each column has five numbers, randomly chosen, between 1 and 15 in the first column, between 16 and 30 in the second column and then 31 to 45, 46 to 60 and 61 to 75, like the instance shown in the third picture.

2	17	32	47	61
4	21	35	51	64
8	22	36	55	66
12	25	41	48	69
13	29	44	59	72

There can be many players, dozens, in fact and each has a card with different numbers, randomly chosen. The person conducting the game then draws numbers, from "1" to "75", at random, from a box and calls them out. And players who find the numbers called out in their cards, score those numbers out. The winner is the one who first scores out all the numbers in any row or any column.

We can see that if we draw numbers at random and score out numbers in our own card, when the numbers are drawn, we are equally likely to first score out a whole row or a whole column. But the remarkable thing, the paradox, the article says, is that when a group is playing the game, the chance of the winner scoring out a row is twice that of completing a column!

In the analysis of the game, the author first looks at a case of row or a column getting scored out within the very first five draws. For this to happen, for a row, once the first number is called, the second number has to belong to a different column. The second number should hence be one of the 60 numbers belonging to other columns, out of the 74 numbers that are left. The chance of this happening is 60/74. Similarly, the chance of the third number belonging to one of the three remaining columns is 45/73. And then, for the fourth and fifth numbers, it is 30/72 and 15/71.

The product of these four probabilities is

$$\frac{60}{74}\times\frac{45}{73}\times\frac{30}{72}\times\frac{15}{71}=0\cdot044$$

which is to say that it is pretty unlikely.

In the same way, we can work out the chance for a vertical column as:

$$\frac{14}{74}\times\frac{13}{73}\times\frac{12}{72}\times\frac{11}{71}=0\cdot00087$$

Scoring out a vertical column is thus 50 times less likely.

This is when we think of the game being completed in the first five calls. From the probability, which is just over 0.04, this may happen only once in about 22 times the game is played. The author then works out what it would be like if the game carried on to the sixth call. Here, the calculation becomes complex, as there are different combinations, basically related to which of the first five calls did not result in scoring out a number in the winning combination. But the result is that the winning chances of both the rows and the columns increase, but the ratio falls - the row is now only about 23 times more likely to win than a column. And in this way, the chances of a row being filled before a column reduces, till the 13th call, when rows and columns are equally likely, and thereafter, if the game should go on, the columns become more likely. The game is most likely to be solved at the ninth call and the game must get solved within 17 calls!

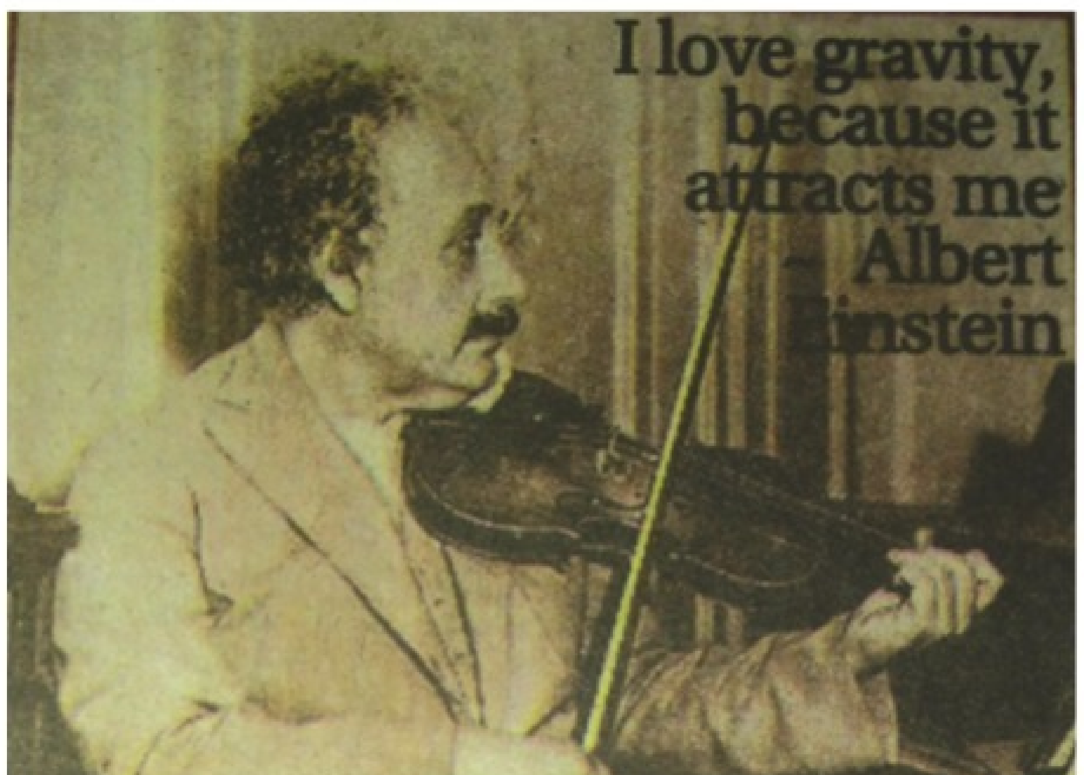
The other articles in the book deal with contemporary issues, like scientific discovery by using information technology, whether brute force methods, which use computers' muscle, can solve all kinds of problems, the question of how mathematics shows up in so many aspects of nature, how Babylon was ahead of the Greeks in Trigonometry and methods of math pedagogy. The first chapter of the book, which speaks about the motivation and the value of math, makes a point that the kind of people who do math may take us by surprise. And the happy idea that there is a mathematician lurking in most of us.

The writer can be contacted at response@simplescience.in

Is there an extra dimension?

Physicists hope that a 'grand unified theory' will bring together strong, weak, and electromagnetic interactions

BINAY MALAKAR



In nature, there are four different types of fundamental interactions or forces: gravitational, electromagnetic, weak and strong interactions.

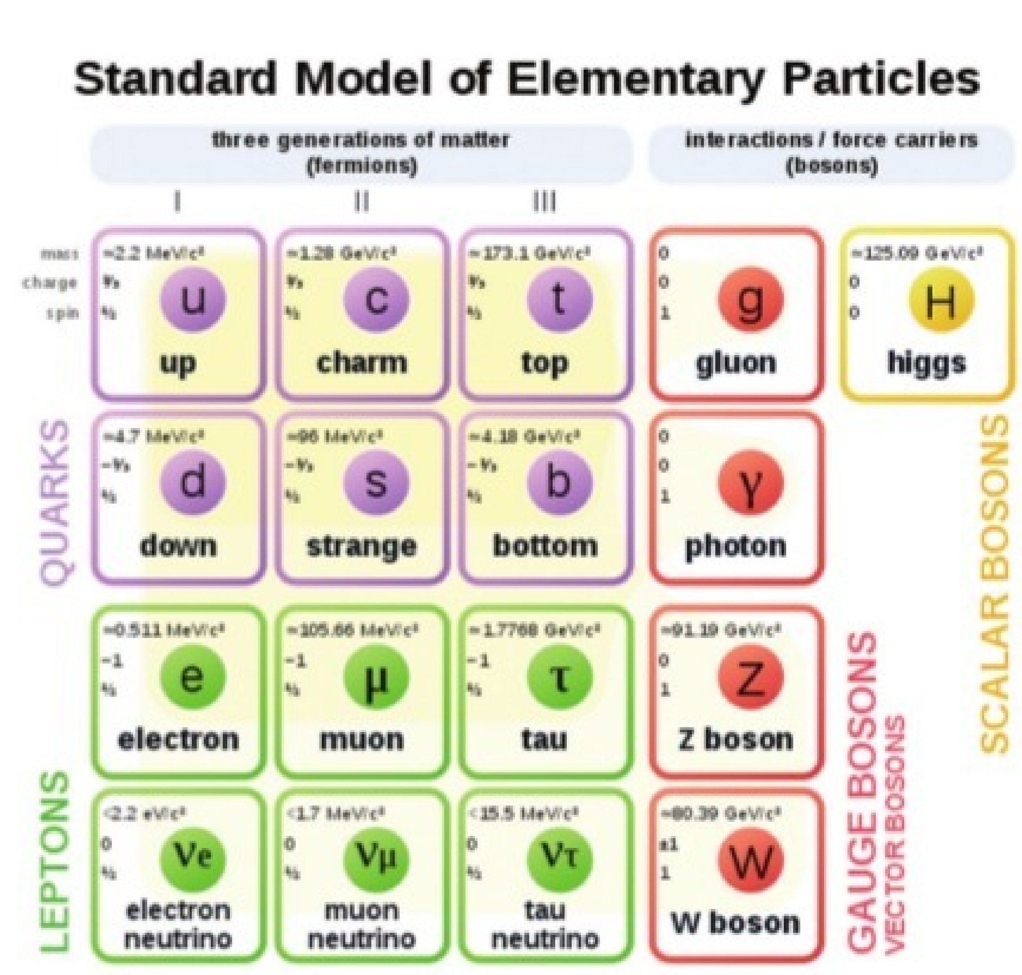
Gravitational interaction is the weakest of all the fundamental interactions and acts between all bodies having mass and is described by the long-range inverse square type Newtonian law of gravitation. This interaction is believed to be mediated through the quantum of interaction -graviton - which is yet to be discovered and provides a large attractive force between the planets and other heavenly bodies. The force that binds our Milky Way galaxy and any other galaxy together is the same as that which holds Venus (or any other planet) in orbit and you to earth - gravitational force. This force is also responsible for the formation of the black hole where not even light (or photon) can escape. The riddle of how black holes and neutron stars are formed, supernovae explosions and many such other problems may be solved easily if we can completely 'identify' gravitational interaction.

Electromagnetic interaction is the unification of electric and magnetic fields. It is much stronger (1036 times) than gravitational interaction and is described by long-range inverse square type law: Coulomb's law. This force arises due to charges of particles and their motion. Electromagnetic force is mediated through the exchange of photons which are massless particles. This field is described by quantum electrodynamics (QED) and is responsible for atomic structure, chemical reactions and other electromagnetic phenomena. This force may be attractive or repulsive.

The third fundamental interaction is the weak nuclear interaction and the strength of weak interaction is 10-10 times that of the electromagnetic interaction. The weak nuclear force is the second weakest force, after gravity. The beta decay of radioactive nuclei, decays of strange particles and nuclear fusion in the sun are some examples of weak interaction. Unlike the previous two interactions, weak interaction is a very short range force and is mediated through massive W± and Z0 bosons. In fact, weak interaction takes place only at very small, subatomic distances. Weak force is not symmetrical under parity transformations. The theory that describes the weak interaction is called quantum flavourdynamics (QFD).

Strong interaction is the strongest interaction in nature and occurs between a neutron and a neutron or a neutron and a proton or a proton and a proton. This is a short range, charge-independent and attractive force. This force is mediated through the exchange of gluons which are massless. In true sense, quark-quark (quarks are the ultimate building blocks of matter) interaction is believed to be mediated through the exchange of gluons. This force is responsible for binding together the fundamental particles (quarks) of matter to form larger particles (neutron or proton). Strong interaction is described by quantum chromodynamics (QCD) and so strong force is also called color force.

Physicists hope that a "grand unified theory" (GUT) will unify the strong, weak, and electromagnetic interactions. There have been several proposed unified theories (Standard Model, String Theory, SO (10) etc.) but we need data to pick which, if any, of these theories describes nature correctly. The discovery of Higgs boson and other such recent discoveries will get physicists closer to knowing which GUT is correct. The above three forces are responsible for all of the pushes and pulls in the universe. If gravity is also combined with these three forces, then GUT will become the proposed "theory of everything" (TOE). GUT is thus an intermediate step towards TOE. If a grand unification of all the interactions is possible, then all the interactions we observe are all different aspects of the same unified interaction. Current data and theory suggest that these varied forces merge into one single force when the particles are at ultra high energy (1016 GeV, 1GeV=109 eV; 1eV = energy needed to bring an electron through a potential difference of 1 Volt). This energy/temperature prevailed only at the time of Big Bang explosion. The problem is that the unification energy is so high that we cannot conceive how to build a particle accelerator to achieve this energy.



The standard model of particle physics is the theory describing three of the four known fundamental forces (electromagnetic, weak and strong interactions except gravitational force) in the universe, as well as classifies all known elementary particles into two groups - fermions and bosons. Standard model was developed in stages throughout the latter half of the 20th century, through the works of many scientists around the world, with the current formulation being finalised in the mid-1970s upon experimental confirmation of the existence of quarks. Since then, confirmation of top quark (1995), the tau neutrino (2000) and the Higgs boson (2012) have added further credence to the standard model. Although the standard model is believed to be theoretically self-consistent and has demonstrated huge successes in providing experimental predictions, it leaves some phenomena unexplained and falls short of being a complete theory of fundamental interactions. It does not incorporate the theory of gravitation as described by Einstein's general relativity or account for the accelerating expansion of the universe as described by dark energy. The model does not contain any viable dark matter particle that possesses all of the required properties deduced from observational cosmology. It does not incorporate neutrino oscillations and their non-zero masses. It is very interesting to note that 96% of our universe is missing or invisible (72% is dark energy and 24% is dark matter). The theory of super symmetry (SUSY) suggests that all known particles have, as yet detected, 'super partners' and these super symmetric particles may help explain one mystery of the universe - missing matter and energy.

According to some physicists the problem of grand unification may be tackled with string theory which basically replaces all the elementary point-like particles that form matter and interactions with a single extended object of vanishing length. The length scale should be comparable to the Plank length (10-15 metre). Accordingly, every known elementary particle, such as an electron, photon, neutrino, quark etc. corresponds to a particular vibration mode of the string. Since string theory incorporates all of the fundamental interactions, including gravity, many physicists hope that it may fully describe our universe, making it a theory of everything In fact, string theory is a promising candidate for quantum gravity also. The current quest for a unified field theory is mainly focused on superstring theory.

Many physicists believe that the universe has more dimensions than the four (three space and one time) we are aware of. The introduction of extra dimension may probably solve the problem of unification, but how many dimensions would be needed to unify modern particle physics with gravity is the current topic of modern research for particle physicists. Large Hadron Collider (LHC) experiment may allow us to see evidence of these extra dimensions. One explanation for gravitational force to be so many orders of magnitude weaker than the other three forces may be that our universe is part of a multidimensional reality and that gravity can leak into other dimensions, making it appear weaker.

For over a century, unified field theory remains one of the major unsolved problems in physics. However, physicists want to find a single theory that describes the entire universe, but to do so they have to solve some of the hardest problems in physics. For decades, confident physicists have said that unification is just around the corner. The discovery of extra dimensions would herald the first change in our view of space-time since Einstein's theory of relativity. It may also solve some longstanding problems in physics and astrophysics. Solving these problems would be a fitting tribute to Einstein, the first superstar in physics who actually started this revolution.

The writer is associate professor and head of the department of physics, Netaji Mahavidyalaya, Hooghly

PLUS POINTS

Supporting primitive life

A planet near our own could support primitive life, scientists have announced.

The nearby world— orbiting around Barnard's Star, the second closest star system to our own is known as a super-Earth and has the potential to serve as a home for simple alien life, researchers have said.

The planet is probably very cold, at minus 170 degrees centigrade. But it could be a better home than we had realised when it was discovered: if it has a big iron and nickle core and geothermal activity, as scientists predict, it might be able to harbour primitive life.

That is according to Villanova University astrophysicists Edward Guinan and Scott Engle who announced their findings at a major meeting of astronomers.

The researchers suggest that the planet, known as Barnard b, might be surprisingly habitable as a result of the processes that are happening underneath its surface.

Scientists announced the discovery of Barnard's b in November, but it was described as being very hostile to any possible life: it is an icy desert, without liquid water, where the sun shines only dimly and the temperature is cold enough to kill any life that might flourish. But the suggestion of heat flowing from underneath its surface could make it a better home than they thought.

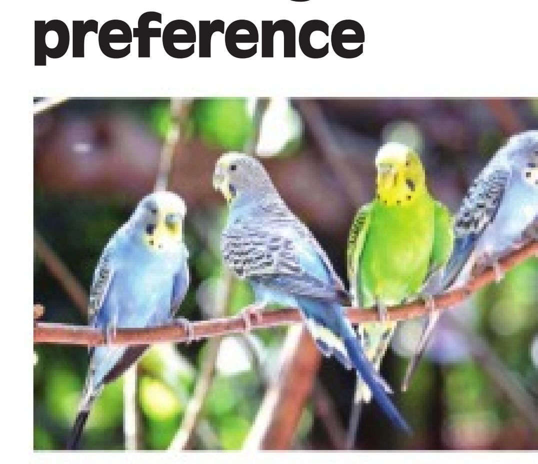
The planet has a mass just over three times that of the Earth, and goes around its star every 233 days, at roughly the same distance that Mercury orbits the Sun.

Researchers now hope they can look more at the planet in an attempt to understand its "atmosphere, surface, and potential habitability". It is very faint in the sky, but future large telescopes could allow us to get a better look at it.

Whatever they find in the future, the planet has already shown us that such planets might be more common than we'd realised, suggesting that yet more of them are waiting to be found.

The independent

Affecting mate preference



Male birds are often the ones with the most vibrant feathers or the most elaborate songs, but researchers recently said that what female birds really appreciate is a male who shows his intelligence.

The report in the journal Science aligns with one of Charles Darwin's old theories, which held that mate choice could contribute to the evolution of intelligence.

Researchers used 34 small Australian parrots, known as budgerigars, to test the notion that a suitor's smarts could outweigh style or songs.

A female bird was exposed to two similar looking males, in a cage in which she could interact with only one at a time. Prior study designs like this have shown that females tend to lean toward males with slightly nicer appearances or more appealing songs.

Researchers could tell which male was preferred by the amount of time the female spent interacting with him. Then, they swept away the lesser male to engage him in a special training session in opening a container filled with seeds. The female and her preferred male received no such training, and were given open boxes of seed to eat from freely.

Next, the female was placed in a cage and was allowed to watch the trained male open his sealed box of seed. She also watched the untrained male. After that, eight of the nine females changed their minds and began spending more time interacting with the more capable box-opening male.

But experts caution that the notion is difficult to study in the animal world, particularly when complex behaviours like mating rituals are in play.

The researchers "offer convincing evidence that female budgerigars modified their mate preference in favour of trained males after observing them perform complex foraging tasks", wrote evolution experts Georg Striedter and Nancy Burley of the University of California, Irvine, in an accompanying Perspective article in the journal Science.

The straits times/ann