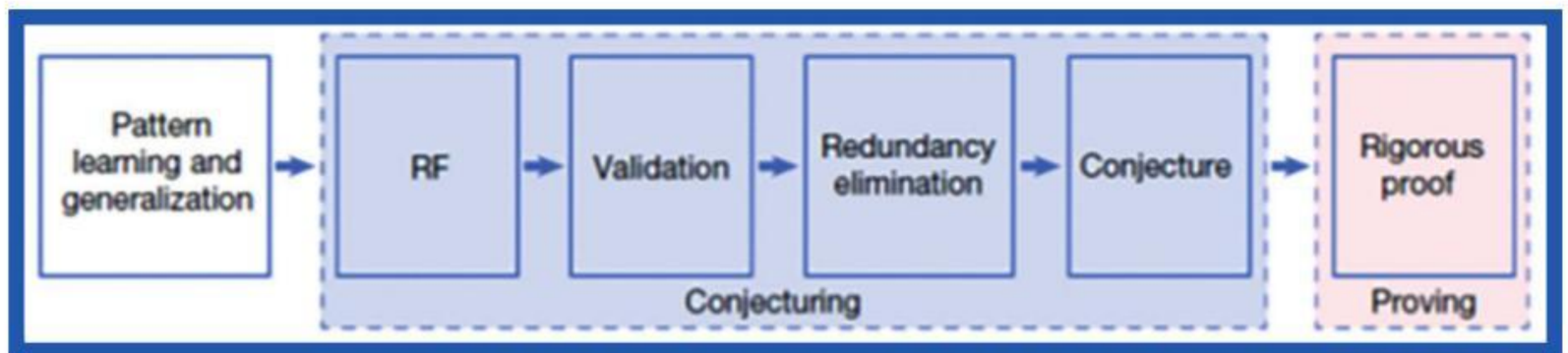



# The Ramanujan robot

## Can the spirit of mathematics be mechanised?

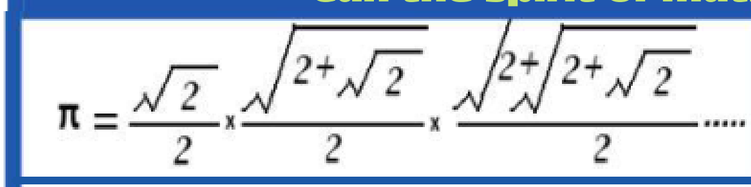


Fig A

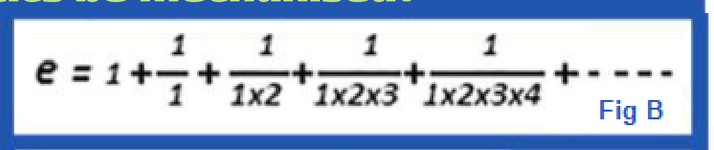


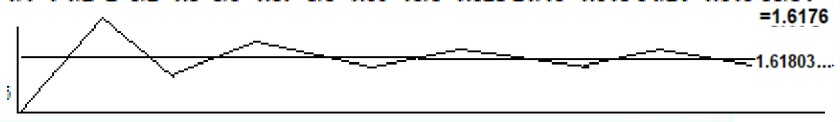
Fig B

**Fibonacci series** In the series: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, each number is the sum of the previous two.

**And if we consider the ratio of each number to the previous one:**

1 1 2 3 5 8 13 21 34 55

1/1=1 1/2=2 3/2=1.5 5/3=1.67 8/5=1.60 13/8=1.625 21/13=1.615 34/21=1.619 55/34 =1.6176



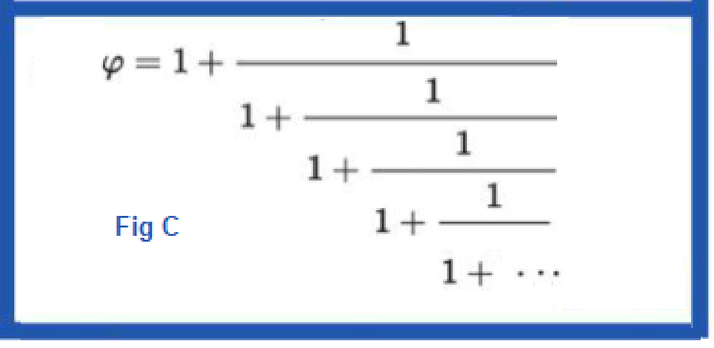



Fig C

**S ANANTHANARAYANAN**

Computers and "big data" have transformed the nature of research in the sciences. Even in the arts, musicians, graphists and architects rely on computerised support. The mechanised input, however, is in the *nature* of "support". The insight that is needed to sift data and see hidden patterns, or to add the creative touch to a composition, remains with the scientist or artist.

Gal Raayoni, Shahar Gottlieb, Yahel Manor, George Pisha, Yoav Harris, Uri Mendlovic, Doron Haviv, Yaron Hadad and Ido Kaminer, from Technion – Israel Institute of Technology, and the Technion Harry and Lou Stern Family Science and Technology Youth Center, Haifa, in a paper in the journal, *Nature*, describe a possible way to automate even this genius, of the mathematician. They describe a method for computers to discover, from a mass of numbers, relationships that fit into formulae – to form the starting point of research into study of how these formulae arise and work.

Over the centuries, gifted mathematicians have identified cases of special numbers which have remarkable properties. And these properties, in turn, reveal more relationships and features, and then, could lead to applications. A well-known example is the number, pi, which is the ratio of the circumference of a circle to its diameter. While this ratio is usually known as 22/7, or 3.14, this value is approximate. A better value is 3.14159, and even better is 3.141592653589793238. But the special feature of pi is that the ratio can never be exactly stated – not because of limitations of measurement, but for reasons of its very nature. Archimedes is credited with an early method, where many-sided straight-line figures (polygons) are drawn touching the inside and the outside of the circle. By increasing the number of sides, the shapes get closer to the circle itself, and the circumference can be approximated. And always improved by adding sides to the polygon!

Similar to the case of pi, we have another number, known as "e", which is related to things that grow like a deposit that bears compound interest. Now, if the interest on the deposit is applied not once a year, but every six months, the interest accumulates a little faster. We can keep reducing the intervals of interest, to once a month, every day, every second, and so on. When the interval is very nearly zero, we find that in the time the deposit would normally have doubled, it reaches 2.71 times its value. This number, again, is not exact, a better value is 2.718281, and so on, and the exact value can never be worked out.

The number "e" has found application in many areas of science and engineering. One such is how fast the radioactivity of a sample would fall, while the number of atoms that are yet to decay keeps reducing. And in other areas, like working with waves, radio waves, light or lasers, atomic and nuclear science, biology and lately, epidemiology, the concept of "e" has become fundamental.

Yet another ratio, of the kind of "pi" or "e" is the "golden ratio", which is the ratio of successive numbers in a series where each number is the sum of the previous two numbers. As we can see in the picture, this ratio progresses to be nearly 1.61803..., but can never be exactly evaluated. This, again, is a ratio that is found in leaves, shells, and many other places in nature.

Mathematicians, who had understood the way of numbers, experimented to discover these and other numbers and then formulae and expressions for calculating them.

Pi, for instance, is given by Figure A, where the value of pi was expressed in terms of continued square roots of two, and no reference to circles. The value of e is given by Figure B. And then, the Golden Ratio was found to be given by the relation in Figure C.

Remarkable relations were even found to exist between some of these numbers. One of these, that connects the numbers, pi, "e" and "i", the imaginary square root of "-1", a relation named after the mathematician, Leonhard Euler, is like this – $e^{i\pi}+1=0$ ).

Many of the relations were, at first, just conjectures, later verified and finally proved. One such is the celebrated Fermat's theorem of 1637, which says that the Pythagoras relation, $a^2 = b^2+c^2$, can never be true if a, b and c are integers and raised to a power more than two. The relation was laboriously verified, again lately with the help of computers, but it defied proof for nearing four centuries.

### Ramanujan

It is by way of conjectures like these that many important advances in mathematics have come about. The mathematician, Srinivasa Ramanujan (1887-1919) was a prolific contributor – identifying a host of spectacular relations, theorems, mathematical results, even in his short working career. Ramanujan had not been trained in the formal proposition-axioms-proof-tradition of mathematics, but was a dropout, yet passionate about mathematics. Many of the results he put down are hence abruptly stated, without the context, discussion or proof. Deciphering his diaries, verifying the startling results that he records, and working out the proofs has been fodder for generations of researchers and students.

The authors of the paper in *Nature* observe that the many fundamental constants, like pi or "e" have become ubiquitous in the sciences, but mathematical formulas that demonstrate links among them are hard to come by. There is regular progress in mathematics and refinement of existing knowledge, but startling discoveries that set off new courses of research arise sporadically, as results of ingenuity or profound insight of mathematical greats, like Carl Friedrich Gauss or Ramanujan. The authors hence propose making use of algorithms and modern computing potential to scan the nature of different mathematical constants, like pi or "e", and try to fit the data into schemes or formulas – which could then be verified and lead to research of how and why they work. A mechanised programme where "computer algorithms use numerical data to unveil mathematical structures, thus trying to replace the mathematical intuition of great mathematicians," the paper says. As mathematical constants are common in different areas of science, discoveries in the field would have wide application, the paper says. Considering the approach as a rough equivalent of *Deep Blue*, the chess playing computer, doing as well as Garry Kasparov, the grandmaster, the authors christen their approach "the Ramanujan machine."

The method used is akin to using artificial intelligence to fit data to a hypothesis, usually a mathematical expression that can predict the future course of the data. The hypothesis is tweaked at each run of the programme till it begins to fit the data. A similar approach, as shown in the diagram, is to propose, test and optimise a "regular formula", denoted by RF, to arrive at a mathematical conjecture.

The authors report that algorithms have found dozens of well-known formulas, as well as previously unknown ones. Some conjectures were simple to prove, some are yet to be proved, the paper says.

As huge computing resources are involved, the authors have followed the example of worldwide collaborative research programmes and launched a "Ramanujan machine" project, where "the general community can donate computational time to find RFs, propose mathematical proofs for conjectured RFs, or suggest new algorithms for finding them," the paper says.

The writer can be contacted at response@simplescience.in

# SURVIVING DESTRUCTION

## Young stars and their planet-forming discs can be ejected from their birth region, but not all of them just wander off into space

**CHRISTINA SCHOETTLER**

When stars like the Sun form, they are born with tens to tens of thousands of stellar siblings in young star-forming regions. In the night sky, some examples of such regions can be seen even with a small telescope, such as the Orion Nebula Cluster (*in photo*), which is located in the Orion constellation and is around 1,350 light years away from us.

The young stars in these regions are much closer together than the distance from the Sun to its closest neighbour, Proxima Centauri, about four light years away from us. This closeness of stars in star-forming regions increases their chance of interacting with each other, often multiple times. These interactions can on occasion cause stars to be ejected from their birth region altogether and we previously thought that most of those stars just wander off into space and never encounter any other stars or star-forming regions again during their lifetime.

Almost at the same time as the stars are forming, discs of gas and dust also form around these stars. We call these structures protoplanetary discs as they are the locations where planets will or have already formed. The interactions that the stars have while they are in their birth region can also have an effect on these discs. They can reduce the size of the discs, alter their orbits around their parent star or even destroy the discs. The interactions that cause a star to be ejected are often particularly intense/strong, as the star must reach the escape velocity to leave the gravitational pull of its birth region. This is similar to rockets to the Moon having to reach escape velocity to escape Earth's gravity.

We knew from previous research that the ONC had already ejected several of its stars during its lifetime. There have even been examples of these ejected stars retaining their planet-forming discs. In our previous research into this cluster we also found a surprising number of stars that were older than the ONC, so these stars must have been born somewhere else, but had flown through the ONC in the recent past. We wondered if these visitors still had a disc, potentially with planets, surrounding them as this would give us insight into the resilience of the discs to external destructive encounters with passing stars.

One of the main ways of finding protoplanetary discs around young stars is through emission of radiation in the Infrared or Ultraviolet band that is higher than one would expect from this type of star alone. Stars emit radiation across the full range of the electromagnetic spectrum, from high-energy gamma rays to low-energy radio waves. The higher excess emission is caused when the radiation from the star is intercepted by the gas and dust in the disc. Excess IR radiation is produced by the dust in the disc when it is heated by light from the young star. Excess UV radiation is produced when the light emitted from the star interacts with molecules in the gas of the disc, such as H2 (molecular hydrogen).

There are many astronomers and observatories searching for protoplanetary discs and this research allows us to better understand the formation of our own Solar System and determine whether there are more planetary systems like our own. We searched through their databases to see if any had observed our ejected stars in the past.

We found five stars with evidence of a disc that had been ejected from the regions where they were formed. Three of the stars had been ejected from the ONC itself, one has been ejected from an unknown region but is heading towards the ONC and the last one has already interacted with and passed through most of the ONC. This showed us that it is possible for stars to be ejected from their birth region, approach a second region, and emerge from it with an intact protoplanetary disc.

Our research has only been made possible thanks to the exquisite precision of data from the Gaia space telescope, which enabled us to accurately determine where these stars had been, and where they were heading to. The Gaia telescope was launched in 2013 by the European Space Agency and measures the positions and motions of over a billion stars in our Milky Way galaxy.

Even though we have only found five stars with evidence of a disc intact after having been ejected from the ONC, we now know that young stars can run away with their planet-forming discs from their birth region and visit another region. Most of our past visitors to the ONC have never been targets in a search for the presence of a disc, as they are far outside star clusters or other regions where star formation takes place. Our findings motivate us to search for them more specifically to find out how rare these occasions really are.

The more we investigate the early stages of star and planet formation the more surprises are thrown up. Despite the rather hostile environment some stars are born in, we are finding that their planetary systems can survive a multitude of destructive encounters with other stars.

The writer is a PhD student at the department of physics and astronomy, University of Sheffield, UK

**PLUS POINTS**

## Already endangered

Scientists have discovered a miniature chameleon that they believe could be the smallest reptile on Earth. Small enough to stand on a human fingertip, two of the lizards, a male and a female, were discovered in a patch of rainforest in northern Madagascar.

But the species might already be threatened with extinction, the journal *Scientific Reports* said.

Poverty and growing populations have led to the clearing of rainforests to make room for farmed animals in the country in the Indian Ocean. More than 90 per cent of Madagascar's forests have been razed, according to Nasa.

The pair, about the size of a sunflower seed, were found by a German-Madagascan team. The scientists recommended the species be assessed officially for the IUCN red list of threatened species as soon as possible.

Mark Scherz, an evolutionary biologist at the University of Potsdam and study co-author, told *National Geographic*, "It's all good and well to say, 'I really hope that people stop deforesting this forest', but until the economic future of Madagascar changes, there's no hope for any of its wildlife because the people have to eat."

However, Oliver Hawlitschek, a scientist at the Centre of Natural History in Hamburg, said, "The nano-chameleon's habitat has unfortunately been subject to deforestation, but the area was placed under protection recently, so the species will survive."

The male "Brookesia nana", or nano-chameleon, has a body that is only 13.5 mm (just over half an inch) long, making it the smallest of all the roughly 11,500 known species of reptiles, zoologists said. Its total length from nose to tail is nearly 22mm. The female is far bigger, at around 29mm long. The tiny reptile has a projectile tongue, which it uses to nab prey, mostly mites.

Frank Glaw, who was part of the international team of researchers that classified the new species, said the pair were spotted on a mountainside by a local guide. "You really have to get down on your knees to find them," he said. "They are obviously camouflaged, and they move very slowly."

He and his colleagues performed a CT scan of the female and discovered that it harboured two eggs, confirming that it was an adult.

–THE INDEPENDENT

## Overfishing to blame

Global numbers of oceanic sharks and rays have plummeted 71 per cent over the last 50 years, a new study by an international research team has found. The decline since 1970 is primarily due to overfishing, say the researchers of their paper published in January in the scientific journal *Nature*.

One of the study's authors, Peter Kyne of the Charles Darwin University in Australia, said the biology of these species limit how fast they can reproduce and therefore sustain fishing pressure. "These large species mature at a late age, and typically have few young. Some species may reproduce only every second or third year," he said.

It is their strategy to produce fewer but larger offspring that will face fewer potential predators than smaller babies. "Their individual chance of survival is high in the absence of fishing," Kyne said. But humans and their appetite for meat, fins and gill plates are driving these species to extinction.

Commenting on a new Singapore study which had found that many processed and cooked fish products were of threatened elasmobranch species, Kyne said there was overlap between species in his pelagic paper and those detected in the Singapore study.

In 2017, a report by wildlife trade monitoring group Traffic and conservation group World Wide Fund for Nature found that Singapore was the third largest importer of shark fin after Hong Kong and Malaysia. "Using genetic barcoding to detect mislabelled and potentially illegally traded species is an important tool for monitoring, compliance, and enforcement," he added.

Consumers can also help by not eating products made from highly threatened species.

–THE STRAITS TIMES/ANN