Mathematical curiosities

There are surprises that lurk in numbers

S ANANTHANARAYANAN

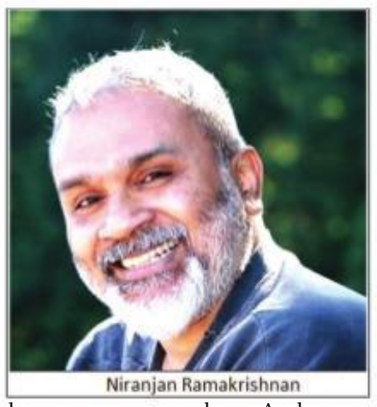
'Mathematics is the language God uses to write the universe," said Galilei Galileo. Galileo may have said this because of the mathematical precision he saw in the way the heavenly bodies moved. And maybe this was the quality Galileo was looking for in the heavens, and happily, he found it.

Numbers, in any case, have been the core of the work of the greatest mathematicians. And it would be good news to many, even those mathematically challenged, that some intriguing properties of numbers are what Niranjan Ramakrishnan, information technology practitioner and polymath, best known for his reflective book on the relevance of Mahatma Gandhi to the contemporary world, has included in much of his extensive blogging. Ramakrishnan had been fighting ALS (Lou Gehrig's disease) since 2009. It was when he started losing his faculties that he took to writing, on a range of subjects, mathematical curiosities being one of them. And his writing has become topical, and many have been writing in, when he recently succumbed to ALS.

One of the ideas Ramakrishnan speaks of is prime numbers, which are numbers that have no divisors, like 3, 5, 7, 23, 31. He notes that all prime numbers, apart from 1 and 3, have to be just one number removed from a multiple of the number, 6. Table 1 displays examples of the earliest primes, and some large ones too.

Ramakrishnan goes on to show why this must be so. For starters, in any group of three consecutive numbers, like 2, 3, 4 or 51, 52, 53 or 130, 131, 132, one of the numbers must be a multiple of 3. We can see that in the first group, we have the number, 3, in the second group, we have the number, 51 and in the third group, the number, 132. And we can see that this must be true for any three consecutive numbers.

Now, if the group of three numbers has a prime as the middle number, for example, the prime, 11, in 10, 11, 12, the middle number can have no factors. And hence, one of the remaining two must be the multiple of 3. Again, as the middle number is a prime, it must be an odd number. This implies that the other two numbers are even numbers. And, as an even number, the number that is a multiple of 3 is also a multiple of 2 x 3, which is 6. And hence, any prime has to come before or after a multiple of 6!


Niranjan Ramakrishnan

In one of his blogs, Ramakrishnan noticed a pattern about the number 25 (and later found that it was there with other numbers too). Let us start with the squares of pairs of numbers evenly on either side of the number, 25. For example, 20 and 30, are on opposite sides of 25, and equally distant from 25. Other pairs are 21 and 29, or 22 and 28, and so on, as in Table 2.

The first pair is 24 and 26, whose squares are 576 and 676. We can see that the last two digits of the squares are the same (76) and the difference of the squares is 100. In the case of the second pair, 23 and 27, also, the last two digits of the squares are the same and they differ, this time, by 200. This pattern holds for other pairs of flanking numbers, down to 12 and 38, or 5 and 45, even negatives, -2 and 52, with squares of 4 and 2,704, differing by 2,700.

Ramakrishnan goes on to note that this is a property of any number, with respect to flanking numbers at the same distance away from the centre, or the "fulcrum", like the middle of a see-saw. And he works it out that the differences are in multiples of 100 because 25 x 4 = 100. If the fulcrum had been some other number, 27 for instance, the differences would have been multiples of 27 x 4 = 108.

To try it out -- if the fulcrum is 27, the first neighbouring pair is 26 and 28, and the squares are 676 and 784, which differ by 108. If we consider the pair, 25 and 29, the squares are 625 and 841, which differ by 216, and so on. "Now, it's easy to mentally reckon squares of numbers," Ramakrishnan's blog says. For example, to get the square of 182, if we use 100 as the fulcrum, the partner number is 18. And 182 squared is 82 x 400 + 18 squared, or 32800 + 324, or 33124.

Table 1

The prime	Corresp. multiple of six	The prime	Corresp. multiple of six	The prime	Corresp. multiple of six	The prime	Corresp. multiple of six
5	1 x 6 - 1	17	3 x 6 – 1	31	5 x 6 + 1	9,311	1,552 x 6 - 1
7	1 x 6 + 1	19	3 x 6 + 1	37	6 x 6 + 1	19,391	3,232 x 6 – 1
11	2 x 6 - 1	23	4 x 6 – 1	41	7 x 6 – 1	199,933	33,322 x 6 + 1
13	2 x 6 + 1	29	5 x 6 - 1	43	7 x 6 +1	999,331	166,555 x 6 + 1

Table 2

Left of 25	Right of 25	Difference
24^2=576	26^2=676	100
23^2 = 529	27^2=729	200
22^2 = 484	28^2=784	300
21^2 = 44291	29^2=84	400
20^2 = 400	30^2=900	500
12^2=144	38^2=1,444	1,300
5^2=25	45^2=2,025	2,000
-2^2=4	52^2=2704	2,700

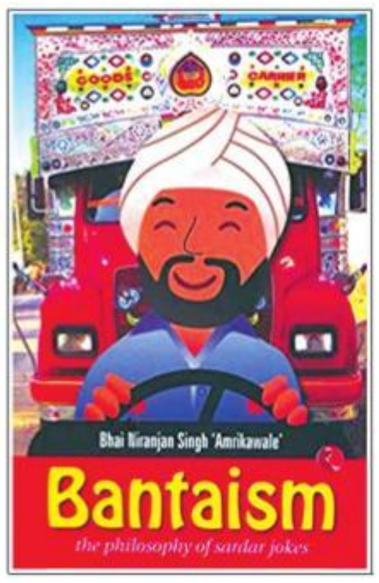


Table 3

Start with 5831			Start with 3682			Start with 2954		
Descending	Ascending	Difference	Descending	Ascending	Difference	Descending	Ascending	Difference
8531	1358	7173	8632	2368	6264	9542	2459	7083
7731	1377	6354	6642	2466	4176	8730	0378	8352
6543	3456	3087	7641	1467	6174	8532	2358	6174
8730	0378	8352	7641	1467	6174	7641	1467	6174
8532	2358	6174						
1467	7641	6174						

Another number that caught Ramakrishnan's fancy was "Karpekar's constant". This is the number, 6174, which arises from any four-digit number. The method is to take the four-digit number and arrange the four digits in ascending order, and then in descending order. For instance, if we start with 4728, we create two numbers, 8742 and 2478. Now, subtract the smaller from the larger. 8742 – 2478 is 6264. Now, repeat the procedure -- the two numbers are 6642 and 2466, and the difference is 4176. When we repeat the procedure, 4176 leads to 1467 and 7641, resulting in 6174. And, now that we are at 6174, we find that the procedure creates no other numbers, for 6174 leads to 1467 and 7641 and the difference, it is again 6174 (please see Table 3, which has other starting numbers).

Now, this number, 6174, has other interesting properties. One of these is that it is divisible by the sum of its own digits: 6 +1 + 7 + 4 = 18. And 6174 = 18 x 343. Such a number, that can be divided by the sum of its digits, is called a "Harshad number" or a "Niven number". And another property of 6174 is that it can be built up by the sum of powers of 18: $18^3 + 18^2 + 18^1 = 5832 + 324 + 18 = 6174$. The discoverer, incidentally, of this and Harshad numbers, was D R Karpekar, a schoolmaster in Nashik, Maharashtra, who was a formidable mathematician. Ramakrishnan, of course, would not rest after reading Karpekar, he went on to check if there were such numbers based on a three-digit starter, and then to generalise to other starters!

As a practicing professional in the world of computer and software, this facility and fascination with numbers found ample expression. But Ramakrishnan's interests were wide-ranging indeed. An important part of his work, different, but not so far, from Gandhi and mathematics, is the two-volume set of *Bantaism*, a compilation of "Sardar jokes" with sensitive and philosophical interpretations of the genre, as illustrative of the foibles of all of us.

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SENDING A MESSAGE ACROSS THE STARS

Blasting out Earth's location with the hope of reaching aliens is a controversial idea -- two teams of scientists are doing it anyway

CHRIS IMPEY

If a person is lost in the wilderness, they have two options. They can search for civilisation, or they could make themselves easy to spot by building a fire or writing HELP in big letters. For scientists interested in the question of whether intelligent aliens exist, the options are much the same.

For over 70 years, astronomers have been scanning for radio or optical signals from other civilisations in the search for extra-terrestrial intelligence, called Seti. Most scientists are confident that life exists on many of the 300 million potentially habitable worlds in the Milky Way galaxy. Astronomers also think there is a decent chance some life forms have developed intelligence and technology. But no signals from another civilisation have ever been detected, a mystery that is called "The Great Silence."

While Seti has long been a part of mainstream science, Meti, or messaging extra-terrestrial intelligence, has been less common.

I'm a professor of astronomy who has written extensively about the search for life in the universe. I also serve on the advisory council for a non-profit research organisation that's designing messages to send to extra-terrestrial civilisations.

In the coming months, two teams of astronomers are going to send messages into space in an attempt to communicate with any intelligent aliens who may be out there listening.


Centre of the Milky Way

These efforts are like building a big bonfire in the woods and hoping someone finds you. But some people question whether it is wise to do this at all.

The history of Meti

Early attempts to contact life off Earth were quixotic messages in a bottle.

In 1972, the National Aeronautics and Space Administration, or Nasa, launched the Pioneer 10 spacecraft toward Jupiter carrying a plaque with a line drawing of a man and a woman and symbols to show where the craft originated. In 1977, Nasa followed this up with the famous Golden Record attached to the Voyager 1 spacecraft.

These spacecraft -- as well as their twins, Pioneer 11 and Voyager 2 -- have now all left the Solar System. But in the immensity of space, the odds that these or any other physical objects will be found are fantastically minuscule.

Electromagnetic radiation is a much more effective beacon.

Astronomers beamed the first radio message designed for alien ears from the Arecibo Observatory in Puerto Rico in 1974. The series of ones and zeros was designed to convey simple information about humanity and biology and was sent toward the globular cluster M13. Since M13 is 25,000 light-years away, you shouldn't hold your breath for a reply.

In addition to these purposeful attempts at sending a message to aliens, wayward signals from television and radio broadcasts have been leaking into space for nearly a century. This ever-expanding bubble of earthly babble has already reached millions of stars. But there is a big difference between a focused blast of radio waves from a giant telescope and diffuse leakage -- the weak signal from a show like "I Love Lucy" fades below the hum of radiation left over from the Big Bang soon after it leaves the Solar System.

Sending new messages

Nearly half a century after the Arecibo message, two international teams of astronomers are planning new attempts at alien communication. One is using a giant new radio telescope, and the other is choosing a compelling new target.

One of these new messages will be sent from the world's largest radio telescope, in China, sometime in 2023. The telescope, with a 500-metre diameter, will beam a series of radio pulses over a broad swath of sky. These on-off pulses are like the ones and zeros of digital information.

The message is called "The Beacon in the Galaxy" and includes prime numbers and mathematical operators, the biochemistry of life, human forms, the Earth's location and a time stamp. The team is sending the message toward a group of millions of stars near the centre of the Milky Way galaxy, about 10,000 to 20,000 light-years from Earth. While this maximises the pool of potential aliens, it means it will be tens of thousands of years before Earth may get a reply.

The other attempt is targeting only a single star, but with the potential for a much quicker reply. On 4 October this year, a team from the Goonhilly Satellite Earth Station in England will beam a message toward the star Trappist-1. This star has seven planets, three of which are Earth-like worlds in the so-called "Goldilocks zone" -- meaning they could be home to liquid and potentially life, too. Trappist-1 is just 39 light-years away, so it could take as few as 78 years for intelligent life to receive the message and Earth to get the reply.

Ethical questions

The prospect of alien contact is ripe with ethical questions, and Meti is no exception.

The first is, who speaks for Earth? In the absence of any international consultation with the public, decisions about what message to send and where to send it are in the hands of a small group of interested scientists. But there is also a much deeper question. If you are lost in the woods, getting found is obviously a good thing. When it comes to whether humanity should be broadcasting a message to aliens, the answer is much less clear-cut.

Before he died, iconic physicist Stephen Hawking was outspoken about the danger of contacting aliens with superior technology. He argued that they could be malign and if given Earth's location, might destroy humanity. Others see no extra risk, since a truly advanced civilisation would already know of our existence. And there is interest. Russian-Israeli billionaire Yuri Milner has offered $ (United States) one million for the best design of a new message and an effective way to transmit it.

To date, no international regulations govern Meti, so the experiments will continue, despite concerns.

For now, intelligent aliens remain in the realm of science fiction. Books like *The Three-Body Problem* by Cixin Liu offer sombre and thought-provoking perspectives on what the success of Meti efforts might look like. It doesn't end well for humanity in the books. If humans do ever make contact in real life, I hope the aliens come in peace.

The writer is University Distinguished professor of astronomy, University of Arizona, United States. This article first appeared on www.theconversation.com

PLUS POINTS

Devastating effect

Lobster larvae exposed to the electromagnetic field of subsea power cables as eggs are more likely to develop deformities and cannot swim as well, new research suggests.

Marine scientists used a specialist aquarium laboratory at St Abbs Marine Station in the United Kingdom to expose more than 4,000 lobster and crab eggs to a level of electromagnetic field predicted to be equivalent to that experienced near underwater cables. Comparative groups of lobster and crab were not exposed in the research, which involved scientists from Heriot-Watt University in Edinburgh and St Abbs Marine Station.

Researchers found lobsters exposed to the field were three times more likely to be deformed than those which were not, with bent and reduced tail sections the most common deformities, while some had disrupted eye development or puffy and swollen bodies.

They were also three times more likely to fail a vertical swimming test to get to the surface to find food. In contrast, electromagnetic field exposure did not appear to have much impact on crab larval deformities and swimming test success, though they were smaller than those not exposed.

With the expansion of marine renewable energy, the number of subsea power cables is rapidly increasing and researchers said consideration should be given to burying them under the seabed to help shield marine species. Alastair Lyndon, a marine biologist at Heriot-Watt University, said, "One potential solution is to bury the cables in the seafloor. This is already done for many marine renewable developments but can be expensive and difficult to maintain. It will be important to ensure its continued inclusion in the consenting process for future projects.

"We must decarbonise our energy supply, but we must also ensure there are as few unintended consequences as possible."

The scientists said the findings about crabs were "reassuring", but longer-term research is needed. Petra Harsanyi, from St Abbs Marine Station, said, "Exposure to the electromagnetic field made the crab larvae smaller. While that hasn't had an immediate effect, it does show that there's an interference with their development.

"It would be interesting to monitor this over time to see whether these crabs have long-term impairments or increased mortality."

The report was published in the *Journal of Marine Science and Engineering**.

The independent

Glass in concrete

Scientists from Nanyang Technological University in Singapore have 3D-printed a bench from a concrete mixture containing recycled glass that could potentially pave the way for more to be recycled for use in building and construction.

The specially formulated material used for the 40 centimetre tall, L-shaped bench exceeded the minimum strength required when hardened to be used in construction projects like heavy traffic driveways, or 30 megapascals.

Concrete used in construction is mainly a mixture of cement, aggregates like sand and gravel, and water. The new mixture made by NTU is a combination of crushed glass from demolished buildings, water and cement. The team said the crushed glass in its mixture can fully replace the sand in concrete for 3D printing. In construction, 3D printing is an important recent development that has made it faster, cheaper and less labour-intensive.

At a media briefing last week, professor Tan Ming Jen from NTU's School of Mechanical and Aerospace Engineering, the principal investigator of the study, said, "Seventy per cent of glass is made up of silicon dioxide... What our research does is to essentially return the silica found in glass to be reused again as sand in our 3D printing concrete mixture."

The study will be published in the *Journal of Building Engineering* in June.

The straits times/ANN