appeared on 21st Nov 2004

# Numbers don't always count

Even infinite numbers of things that grow smaller as they grow numerous may not amount to much, says S.Ananthanarayanan.

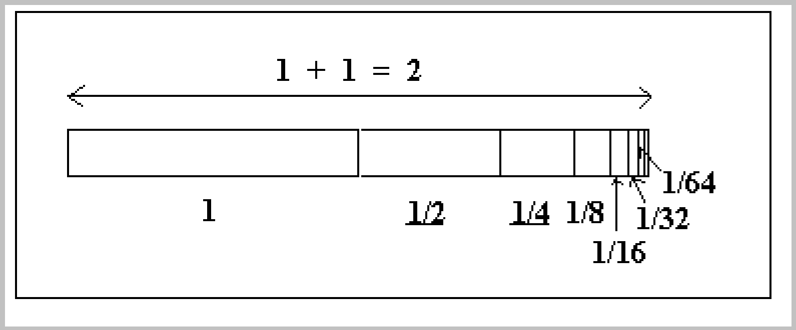
But it is not always easy to work out just how far the total of an infinite list of numbers would reach. In some cases the numbers further down in the list become negligible in value and the total stops growing. In other lists, this fall in values as we go down the list may not be fast enough and the total of series still grows, although the values of the numbers fall. In such cases the list adds up to infinity!

**Converging series**

Take a series like this:

1 + ½ + ¼ + 1/8 + 1/16 + - - - - - - - - - - - - -

If this were an infinitely long list of numbers, and we were to add together the infinite number of terms, should the sum grow so much that it becomes infinite? Or will the value of the later numbers also grow so small that they do not matter? The question becomes easier to answer if we draw it out as a picture:




In the picture, it is clear to see that the later terms in the series get so small that the sum will never cross the number 2. And this number the total will reach after an infinite number of terms. This kind of series is said to *converge to 2*

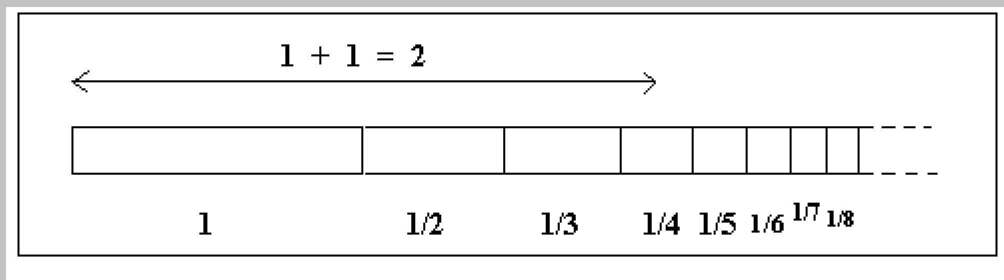
**Diverging series**

Let us take a series like:

1 + ½ + 1/3 + ¼ + 1/5 + 1/6 + - - - - - .

Here also, the successive numbers get smaller and the numbers way down the list should be small indeed. But this is a case where the fall is not *fast* enough and the series does not *converge*.

Pictorially, the series is like this:

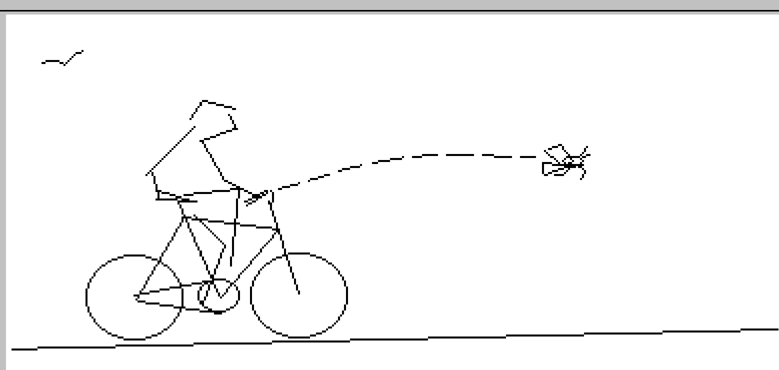



We can see that the terms do not get smaller that fast and the sum does not seem to end anywhere. Take the instance of two successive terms far down in the series, say the 101st and 102nd terms. There is hardly any difference in the values of 1/101 and 1/102. This implies that the successive terms are not getting a lot smaller, like they were in the previous case where every term was only half the value of the previous term.

This series is then a series of long stretches of nearly the same number being added and over infinite terms, the sum is also infinite. This is a case of a divergent series.

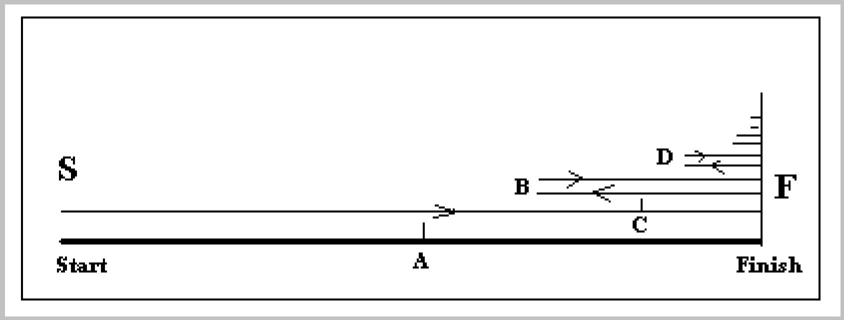
**More complex series**

There is a story about the mathematician Hardy, who was asked this question: A cyclist sets out on a 1-mile trip at a speed of 10 miles an hour.



A housefly on the bicycle handlebar also sets off, at a speed of 20 miles an hour. The fly goes to the end of the mile-long course and then flies back, to touch the bicycle and sets off again.

The fly again reaches the end of the course and back to the bicycle it flies, and so on, till the cyclist himself completes the course. The sequence is shown in the diagram.




When the fly first reaches the end of the run, the cyclist is half-way, at A. When the fly meets the cyclist, he is at B. The fly sets off again to reach the finish when the cyclist is at C, meets him again at D, and so on. The question is, what is the total distance the fly travels?

One way to work it out is to add together the distances from S to F, F to B, B to F, F to D, D to F and so on. The numbers are: 1 + 1/3 +1/3 + 1/9 + 1/9 + 1/27 + 1/27 + 1/81 + 1/81 + 1/243 + 1/243 + - - - - - ,
which is: 1 + 2/3 + 2/9 + 2/27 + 2/81 + 2/243 + - - - - -. Finding a method to calculate the sum of this series would be a complicated exercise that could take time!

But there is another way to tackle the problem, without considering infinite series at all. This is by just considering how long the whole journey lasts. This is the time the cyclist takes to do the 1 mile and the fly is in motion for the same time as the cyclist. As the fly is moving at twice the speed of the cyclist, she would travel for twice the distance. The cyclist travels 1 mile and the fly travels 2 miles!

When the celebrated Hardy was asked this question, he had the answer in a trice. “Of course, Dr Hardy”, the interviewer said, “ you would easily see that there was no need to actually sum the infinite series”. “Really”, asked Dr Hardy, “is there another method?”